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Improved characterization of elastic flexural and shear buckling strengths of I-sections

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Abstract
Design specifications across the world use simplifying assumptions in recommended design equations for structural members. For example, the Eurocode provides section classifications for I-sections subjected to flexure based on ideal simply-supported boundary conditions at the web-flange junctions, while the AISC and AASHTO assume nearly fixed support conditions. Both design specifications consider simply-supported boundary conditions on the web plate in calculating the elastic buckling strength of I-sections in shear. However, the true behavior of I-sections depends on the relative stiffness of the flanges and webs. While simplistic design equations are essential to enable hand calculations or programming in commercial software, it is also important to avoid overly conservative or unconservative estimates of the member strengths. In addition, these simplified assumptions pose problems in interpreting experimental data. This paper looks at the true boundary conditions at the web-flange junctions in I-sections for pure flexure and pure shear using finite element simulations. The impact of the true boundary conditions on the current section classifications for flexure in the European and American work is discussed. This work includes studies on both doubly-symmetric and singly-symmetric I-sections.

1. Introduction

Member design equations recommended in code provisions across the world are based on several simplifying assumptions. While this simplicity is necessary for easy implementation of equations by a design engineer, these assumptions often lead to overly conservative or unconservative design. It is thus important to recognize the possibility of such occurrences in the typical design space, by understanding the true mechanistic behavior of steel structural members and address the problem appropriately. This is also essential to properly interpret experimental and finite element (FE) test data. This paper addresses a fundamental issue with respect to the elastic shear buckling and flexural buckling capacities of I-sections.

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The elastic plate buckling equation is given by

$$\sigma_{cr} = \frac{k\pi^2 E}{12\left(1-\upsilon^2\right)\left(D/t_w\right)^2} \tag{1}$$

where, k is the elastic buckling coefficient, E is the Young's modulus of elasticity, v is the Poisson's ratio, and D/t_w is the ratio of clear depth to thickness of the web plates. The elastic buckling coefficient depends primarily on the loading and boundary conditions. The elastic buckling coefficient for shear is henceforth referred to as k_v and the coefficient for a plate subjected to flexural compression (also known as elastic bend-buckling coefficient) is denoted by $k_{flexure}$ in this paper. The clear depth of the web in Eq. (1) (D) is replaced with the value of twice the depth of the web in compression ($2D_c$) while evaluating the web bend-buckling strength.

The ultimate shear strength of an I-section is typically expressed as the sum of its elastic shear buckling capacity and its post-buckling capacity. White and Barker (2008) provide a comprehensive analysis of twelve models for shear resistance of transversely stiffened I-girders by focusing on the correlation of the various shear strength equations with experimental data. However, most models use ideal boundary conditions at the web edges in computing the elastic shear buckling capacity. The shear strength equations in AISC (2016) and AASHTO (2016) are primarily based on the equations recommended by Basler (1961), which assumes simply-supported boundary conditions at the web edges in computing the elastic shear buckling capacity. The Eurocode equations conform to the recommendations by Porter et al. (1975) and Höglund (1997), also assuming simply-supported boundary conditions at the web edges. Chern and Ostapenko (1969), on the other hand, recommend equations based on fixed conditions at the web edges.

Lee et al. (1996) recognized that the elastic shear buckling coefficient, k_v for webs of unstiffened I-sections depend upon the geometric variables of cross-sections, and that the flanges offer restraint at the web edges, the degree of which may vary between the simply-supported and the fixed edge conditions. Similarly, Al-azzawi et al. (2015) examined the combined restraint offered by the transverse stiffeners and flanges to the web plates loaded in shear. However, Azzawi et al. only analyzed web panels with transverse stiffeners on one edge, with the other edge simply-supported.

Similarly, plate buckling theory stipulates that the elastic buckling coefficient for a plate subjected to flexural compression, $k_{flexure}$ is 23.9 for a simply-supported plates and 39.6 when the edges are fully restrained. However, $k_{flexure}$ also depends on the relative dimensions of the flanges and web. The estimate of $k_{flexure}$ affects the estimates of flexural capacities of I-girders in several ways. Firstly, the section classification of the web at the noncompact web slenderness limit (or for class IV sections) is directly impacted by $k_{flexure}$. AISC (2016) assumes nearly fixed restraint at the web edges bounded by flanges, with an effective $k_{flexure}$ of 36. Conversely, Eurocode (CEN 2006) conservatively assumes simply-supported boundary conditions with a $k_{flexure}$ of 23.9. Subramanian & White (2017) show that the true k lies between the simply-supported and fixed values, and primarily depend on the relative areas of the compression flange and the web area in compression. An assumption of 36 (in AISC) or 23.9 (in Eurocode) leads to a wrongful classification of a slender web (class IV) as a noncompact web (class III) and vice versa. Rajendiran and Subramanian (2018) demonstrated experimentally that the true strength of a cross-section may be substantially lower than the AISC recommended value (due to a noncompact classification of a slender web).

Further, Subramanian et al. (2018) discuss the broader implications of this wrongful classification throughout the beam design strength curve, including the effects of lateral torsional buckling (LTB). They showed that a modified web section classification accounting for restraint effects from flanges is one factor in obtaining a greater reliability index, when correlated with experimental test data.

This paper first examines the effect of flanges and transverse stiffeners on the elastic shear buckling coefficient and then attempts to characterize the influence of flange restraint on elastic flexural buckling capacity of I-sections. This is then followed by an analysis of the impact of the restraints at the web edges on the web section classifications in Eurocode (CEN 2005) and AISC (2016). The analyses presented in this paper are from finite element (FE) simulations in ABAQUS (2018). The elastic eigen value analyses are presented for both doubly-symmetric and singly-symmetric I-sections for a wide range of geometric parameters.

2. Finite Element Modelling

The webs and flanges of the I-girder are modelled using S4R shell elements in ABAQUS (2018), while the transverse stiffeners are modeled using the B31 beam element. Mesh convergence studies and validation with closed-form solutions indicate that 20 elements along the web depth, and eight elements across the width of the flanges are sufficient for beams subjected to pure shear. Likewise, 60 elements along the depth of the web, and 12 elements across the flange width are sufficient for the beams subjected to uniform moment. The aspect ratio of elements is approximately one.

The modulus of elasticity, E, of structural steel is taken to be 200 GPa, and Poisson's ratio, ν, is taken as 0.3.

3. Shear Buckling of I-Sections

This section discusses the analyses and behavior of I-girders subjected to pure shear. The loading and boundary conditions used for the FE models are discussed below.

3.1 Boundary Condition and Loading

The beam is modelled with flexural and torsional simply-supported boundary conditions.

The load is applied as shown in Fig. 1, where V is the shear force per unit length. Shear stress of V/b_{f1} and V/b_{f2} are applied on the top and bottom flange, respectively, thereby subjecting the web to a state of pure shear.

3.2 Case Studies

The influence of flanges and transverse stiffeners on the elastic web shear buckling coefficient is first independently analyzed. This is followed by analyses of I-sections with both flanges and transverse stiffeners. These studies are grouped as below:

- Case i: Web plates supported by flanges and no transverse stiffeners
- Case ii: Web plates supported by transverse stiffeners and no flanges

- Case iii: Web plates supported by both flanges and transverse stiffeners

Table 1 lists the cross-sectional parameters used in the finite element simulations. The clear depth of the web panel (D) used in all studies is 2000 mm. In this paper, a is the web panel length between the transverse stiffeners, t_w is the web thickness, b_f is the average flange width, t_f is the average flange thickness, b_s is the transverse stiffener width and t_s is the transverse stiffener thickness.

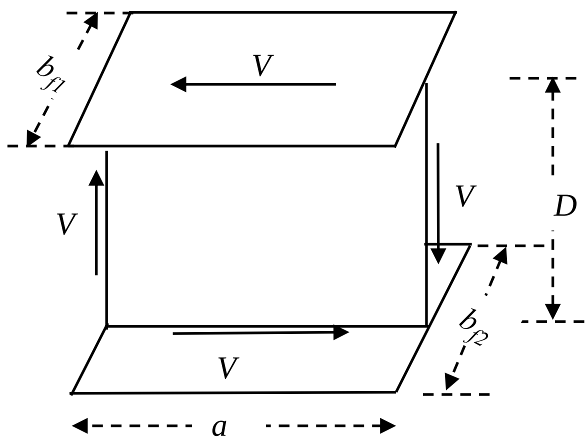

Figure 1: I-section subjected to pure shear

Table 1: List of cross-sectional parameters used in finite element simulations

Cross-sectional parameter	Case i	Case ii	Case iii
No of models, N	493[1]	146	504
a/D	1.0-4.0	1.0-4.0	1.0
D/t_w	70-300	100-300	100-300
D/b_f	4.0-6.0	NA	6.0
t_f/t_w	1.1-5.0	NA	1.1-4.0
D/b_s	NA	6.0-24.0	6.0-24.0
t_s/t_w	NA	0.21-6.5	0.21-6.5

[1] 473 doubly-symmetric girders and 20 singly-symmetric girders

3.3 Influence of Flanges on the Shear Buckling Capacity of Web Panels: Case i

The studies presented in this section clearly indicate that the flanges of I-sections significantly influence the elastic shear buckling capacity of I-girders. Fig. 2 shows the buckling modes for an I-section with web depth-to-thickness, D/t_w of 200, web depth to flange width ratio, D/b_f of four and a panel aspect ratio, a/D of two. The figure also shows the buckling modes for the web panel with simply-supported and fixed boundary conditions in lieu of the flanges for comparison. Four different flange-to-web thickness ratios, t_f/t_w of 1.1, 1.5, 2.0 and 3.0 are shown here. The deformation scale factor corresponding to these figures is 400.

It is seen that the buckled shape of an I-girder subjected to pure shear transitions from that of a simply-supported plate to a fixed-ended web plate with increase in flange thickness. For example, the girder in Fig. 2(c) with a t_f/t_w of 1.1 buckles in single curvature along the web diagonal similar to the simply-supported plate in Fig. 2(a). These girders also exhibit significant flange displacements. Conversely, the girder in Fig. 2(f) with a t_f/t_w of three buckles in double curvature along the panel diagonal, akin to the plate with fixed edges in Fig. 2(b). Intermediate flange thicknesses convey a transition between the two buckling modes. This behaviour is further reflected in the values of elastic shear buckling coefficients (k_v) obtained from finite element analyses. This behavior is consistent with the observations made by Lee et al. (1996).

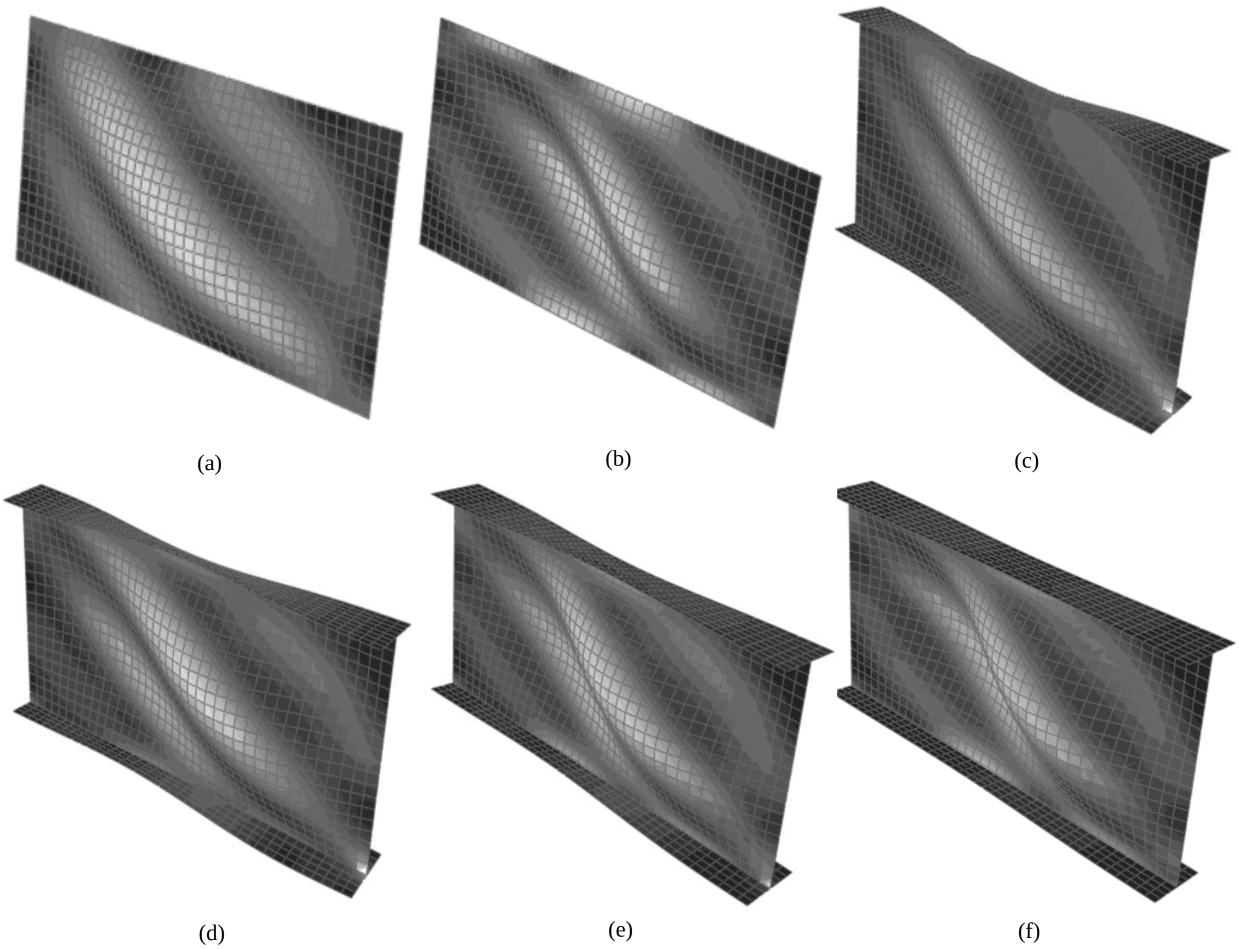

Figure 2: Buckling mode of I-girders for a/D = 2, D/b_f = 4, and D/t_w = 200
(a) S-S plate; (b) F-F plate; (c) t_f/t_w= 1.1; (d) t_f/t_w = 1.5; (e) t_f/t_w = 2; (f) t_f/t_w = 3

The shear buckling coefficient is calculated using Eq. (1), wherein the buckling strength is obtained from the eigen value analyses in ABAQUS (2018). Figs. 3-6 show the variation of k_v with t_f/t_w for different cross sectional aspect ratios (D/b_f = 4, 5, 6) and panel aspect ratios (a/D = 1, 2, 3 & 4), along with the coefficients for a web plate with simple and fixed ends. The following are gleaned from these plots:

1. For a given flange width, an increase in the flange thickness results in an increase in k_v. An increase in t_f/t_w greater than three does not result in an appreciable corresponding increase in k_v, while achieving nearly fixed boundary conditions from the flanges.
2. These plots show that the k_v is not significantly dependent on the web depth-to-flange width ratio (D/b_f) of cross-sections with t_f/t_w greater than three.

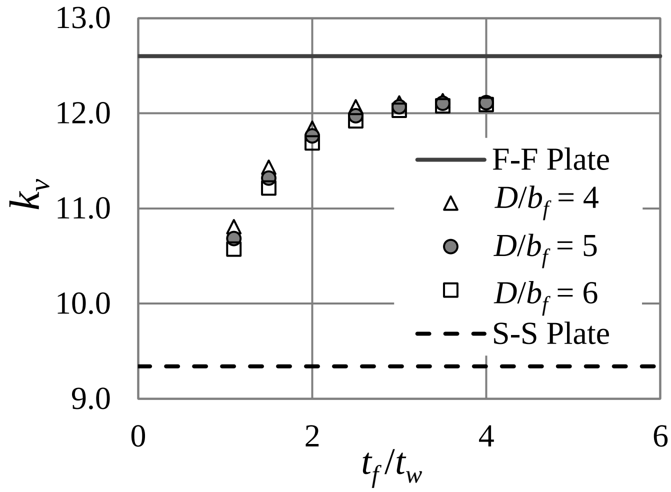

Figure 3: Variation of elastic shear buckling coefficient with the flange–to-web thickness ratios (a/D = 1.0; D/t_w = 200)

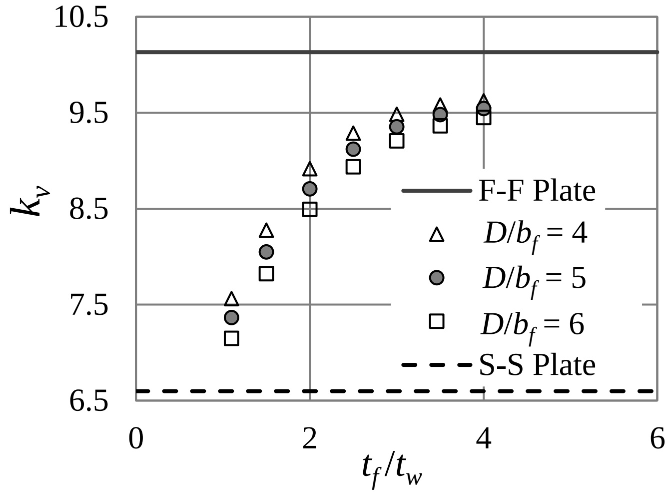

Figure 4: Variation of elastic shear buckling coefficient with the flange–to-web thickness ratios (a/D = 2.0; D/t_w = 200)

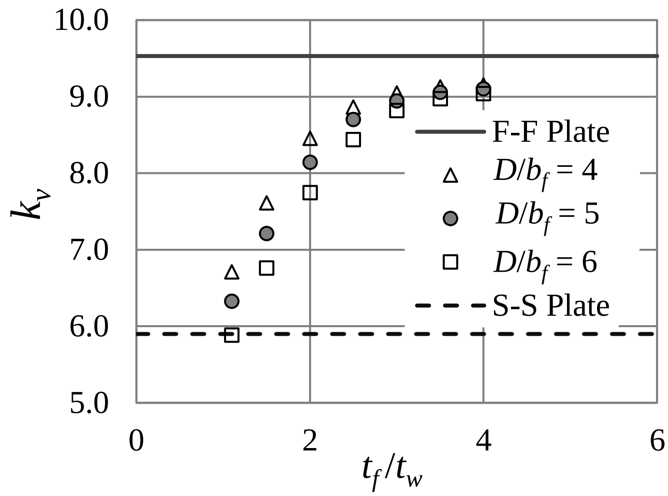

Figure 5: Variation of elastic shear buckling coefficient with the flange-to-web thickness ratios (a/D = 3.0; D/t_w = 200)

3. The elastic shear buckling coefficient also does not significantly depend on the web depth-to-flange width ratio (D/b_f) of cross-sections with small aspect ratios (a/D = 1). However, in girders with large aspect ratios and small flange thicknesses, the effect of D/b_f on k_v is as large as 10%. Neglecting this effect for narrow flange I-girders may be unconservative.

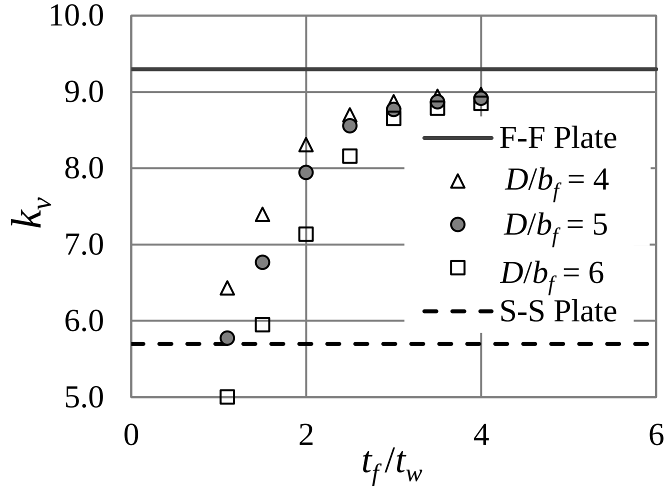

Figure 6: Variation of elastic shear buckling coefficient with the flange-to-web thickness ratios (a/D = 4.0; D/t_w = 200)

Figs. 7 and 8 show the variation of k_v with D/t_w for different t_f/t_w ratios (such as, 1.1, 2.0, 3.0) and a/D ratios such as one and four, respectively. These plots are representative of all the panel aspect ratios studied in this paper. These figures indicate that for a given t_f/t_w, the elastic shear buckling coefficient increases with increase in the web slenderness ratios up to 200. The increment is not significant (< 5%) for a web panel aspect ratio equal to one or for D/t_w > 200, while the difference is as large as 10% for D/t_w less than 150. Based on these studies, it is evident that the girders with thicker webs require proportionately greater flange dimensions in order to develop equivalent amounts of elastic shear buckling strengths as that of slender webs.

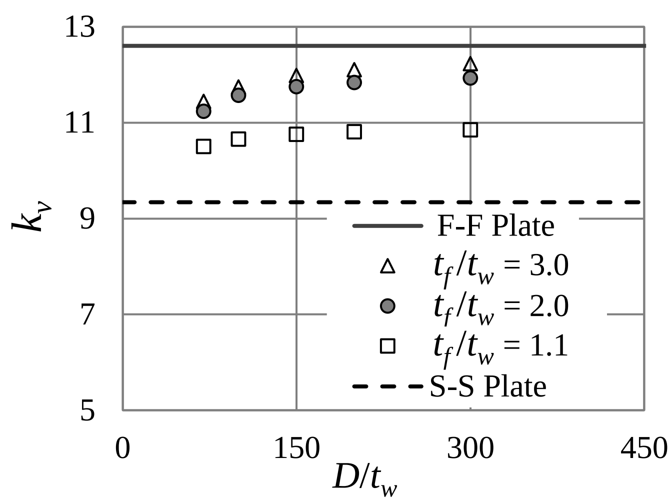

Figure 7: Variation of elastic shear buckling coefficient with web slenderness ratios (a/D = 1, D/b_f = 4)

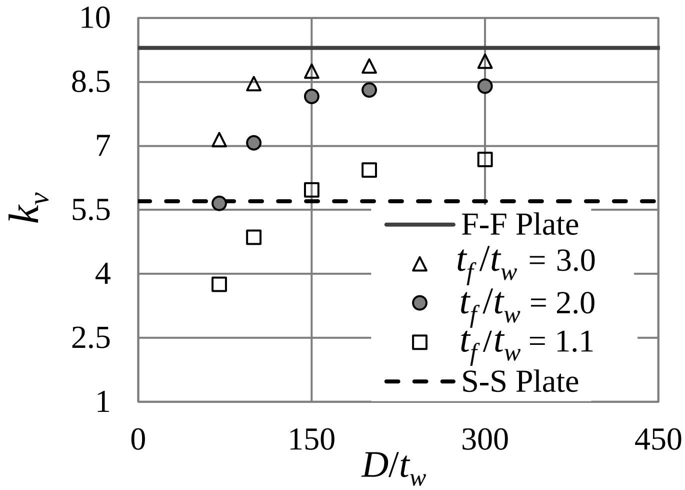

Figure 8: Variation of elastic shear buckling coefficient with web slenderness ratios (a/D = 4, D/b_f = 4)

The elastic shear buckling coefficient is, in some cases, lower than that of a simply-supported plate, as shown in Fig. 8. Table 2 lists the cross-sectional parameters of I-girders, whose elastic shear buckling capacities are less than that of plates with simply-supported boundary conditions. This occurs primarily in I-girders with D/t_w < 150 and t_f/t_w of 1.1. This is because a smaller flange rigidity in the direction normal to the web depth causes excessive displacements of the flanges. This flange lateral movement is common in narrow-flange girders (D/b_f = 6.0). For such I-girders, the assumption of simply-supported boundary conditions in the expressions for elastic shear buckling capacities in Eurocode (CEN 2006) and AISC (2016) may be unconservative by 60%.

Table 2: Cross-sectional parameter (t_f/t_w) of I-girders with elastic shear buckling capacity smaller than that of simply-supported plates

D/b_f	6.0			5.0			4.0	
a/D	2.0	3.0	4.0	2.0	3.0	4.0	3.0	4.0
D/t_w = 70	1.1-1.5	1.1-2.5	1.1-3.5	1.1	1.1-1.5	1.1-2.5	1.1	1.1-1.5
D/t_w = 100		1.1-1.5	1.1-2.0		1.1	1.1-1.5		
D/t_w = 150		1.1	1.1-1.5			1.1		

Twenty singly-symmetric girders with D/t_w = 100, 200 and 300 are analyzed with aspect ratios in the range of one to four. An equal number of doubly-symmetric girders are modeled for comparison, with flange widths equal to the average of the two flange widths used in the singly-symmetric girders. The buckling strengths of the singly-symmetric girders are found to be less than 6% of the doubly-symmetric girders with the same average flange thickness. The difference in the buckling strengths is due to unequal flange stiffnesses at the two web edges, causing unsymmetrical shear buckles in the web plates. This difference however, can be neglected in practical design, and the same equations developed for doubly-symmetric girders may be used for singly-symmetric girders, as also suggested by Lee et al. (1996).

3.4 Influence of Transverse Stiffeners on the Shear Buckling Capacity of Web Panels: Case ii

The improved strength of web plates due to the restraint offered by the transverse stiffeners is presented in this section. These studies are only presented out of academic interest to isolate the influence of transverse stiffeners on the web shear buckling capacities from that of the flanges.

The transverse stiffener dimensions are chosen such that the stiffener areas and moments of inertia conform to the provisions of AASHTO (2016). The ratio of the elastic shear buckling coefficient for a plate with transverse stiffener (k_v) to that of the simply-supported plate ($k_{v,ss}$) is plotted with respect to the stiffener-to-web thickness ratios (t_s/t_w) in Fig. 9, which is a representative plot for the various cases studied. The web slenderness ratio of the models studied in Fig. 9 is 100 and the web depth-to-stiffener width ratio (D/b_s) is six. The transverse stiffeners are seen to greatly enhance k_v for web panel aspect ratios smaller than or equal to one, while providing negligible increase in strength for larger aspect ratios. This observation is instrumental in restricting the panel aspect ratio to one in further studies on the influence of transverse stiffeners on the I-girder elastic shear buckling capacity.

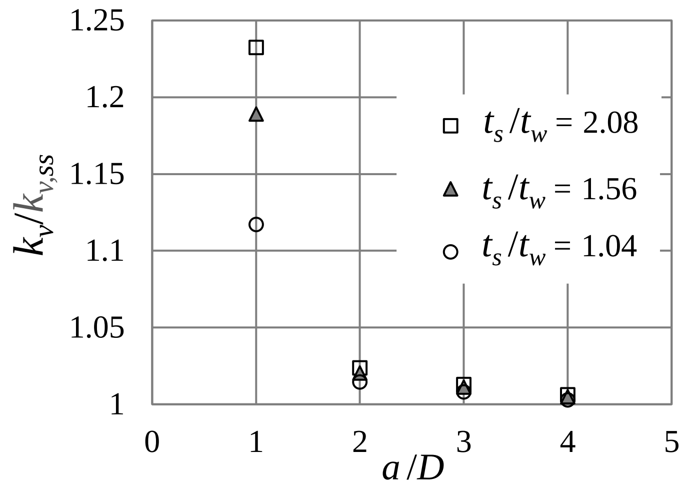

Figure 9: Variation of elastic shear buckling coefficient with a/D (D/t_w = 100, D/b_s = 6)

Fig. 10 shows the influence of t_s/t_w on the elastic shear buckling coefficient obtained for different D/b_s with a web panel aspect ratio of one and web slenderness ratio of 100. Fig. 11 shows the influence of t_s/t_w on the elastic shear buckling coefficient of a panel with a/D equal to one, for different web slenderness ratios studied (such as, 100, 200, and 300). The behaviour of I-girders illustrated by these plots is typical of all the girders studied in this paper. The following points are observed from Figs. 10 and 11.

1. Transverse stiffeners with t_s/t_w less than 0.5 do not affect k_v significantly. The shear buckling coefficient is similar to that of a simply-supported web plate in such conditions.
2. A further increase in transverse stiffener thickness can result in an increase in shear buckling coefficient of up to 33% as compared to that of the plate with simply-supported boundary conditions.
3. Fig. 10 shows that the width of the transverse stiffeners has little influence (< 5%) on the elastic shear buckling coefficient.
4. Fig. 11 shows that the shear buckling coefficient is also little affected by the web slenderness ratio, similar to the observations made in Fig. 7 for web panel aspect ratio of one.
5. These studies indicate that it is highly conservative to ignore the transverse stiffener in the calculation of elastic shear buckling coefficient for girders with aspect ratio less than or equal to one, and an enhancement may be obtained by considering the effect of t_s/t_w.

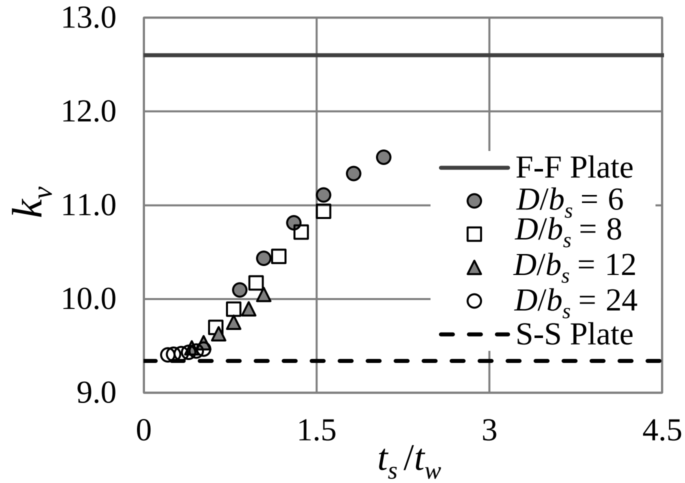

Figure 10: Variation of elastic shear buckling coefficient with t_s/t_w (a/D = 1, D/t_w = 100)

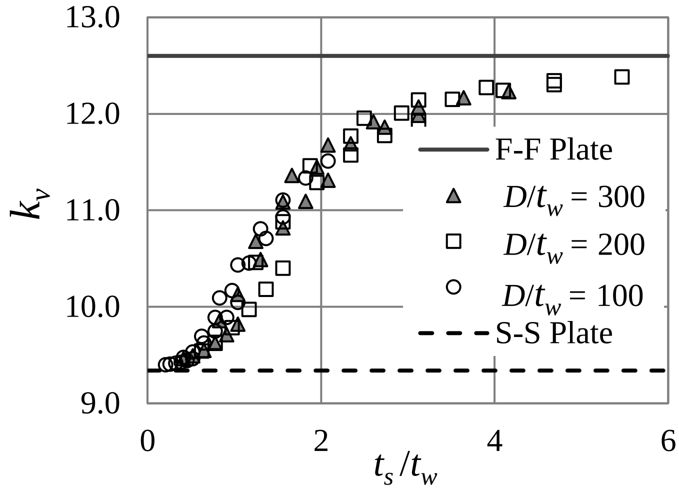

Figure 11: Variation of elastic shear buckling coefficient with t_s/t_w (a/D = 1.0)

3.5 Combined Influence of Transverse Stiffeners and Flanges on the Elastic Shear Buckling Coefficient: Case iii

From Section 3.4, it is evident that the effect of transverse stiffeners on the elastic shear buckling capacities may be ignored for web panel aspect ratios greater than one. The I-girders modelled in case (i), with a/D greater than one, are combinedly studied with an additional 504 stiffened I-girders modelled in this section for a/D of one. The ratios between the shear buckling coefficients obtained for transversely stiffened girders (k_v) and that of the transversely unstiffened girders (k_{vu}) (obtained from the studies conducted in case i.), for web aspect ratio of one are plotted for a range of t_s/t_w in Fig. 12. The girder represented in Fig. 12 has a web slenderness ratio of 100. A web depth-to-flange width of six is chosen for these studies (Fig. 3 indicates that D/b_f does not affect the elastic shear buckling coefficient significantly for panel aspect ratio of one). Fig. 12 clearly shows that transverse stiffeners increase the elastic shear buckling strengths by upto 25% as compared to the values obtained for unstiffened girders.

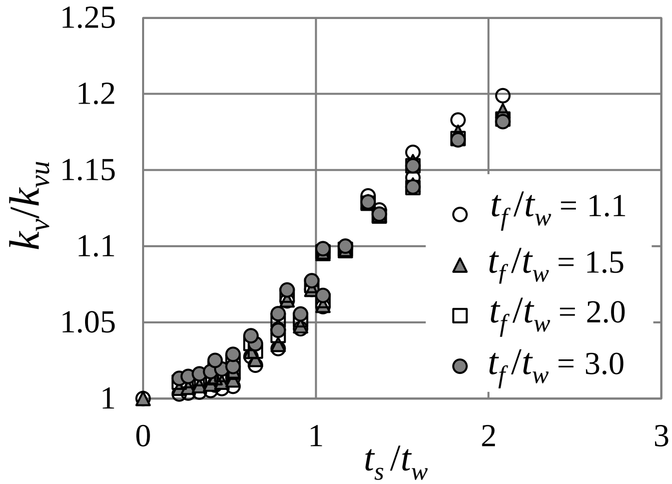

Figure 12: Variation of elastic shear buckling coefficient with t_s/t_w (a/D = 1, D/t_w = 100)

Table 3 lists the statistics obtained for 885 models studied. This shows that k_v estimated using AISC (2016) and Eurocode (CEN 2006) are conservative by upto 71% and 63%, respectively. Lee's equation (Lee et al. 1996) is found to be conservative by upto 30% for stiffened I-girders with panel aspect ratio equal to one. However, it is highly unconservative for greater panel aspect ratios.

Table 3: Comparison of the elastic shear buckling coefficient obtained from FE simulations with empirical equations

Statistics	$k_{v,FEA}/k_{v,AISC}$ (AISC 2016)	$k_{v,FEA}/k_{v,EC}$ (CEN 2006)	$k_{v,FEA}/k_{v,Lee}$ (Lee et al. 1996)
Mean	1.33	1.3	1.03
COV	0.14	0.14	0.15
Min	0.42	0.40	0.34
Max	1.71	1.63	1.30

4. Boundary Conditions at the Web-Flange Junctions in I-Sections for Pure Flexure

This Section discusses the restraint offered by the flanges to the webs in I-girders subjected to flexure, and how this impacts the current web classifications in AISC and Eurocode.

4.1 Loading and Boundary Conditions

The girders are flexurally and torsionally simply-supported (twist is restrained, but the flanges are free to warp at the supports). The test girders are subjected to uniform moment and are provided with sufficient torsional braces to preclude lateral torsional buckling. Furthermore, cross-sectional distortion at the supports due to the load application is prevented by enforcing Vlasov beam kinematics.

4.2 Influence of Web Panel Aspect Ratios on the Bend-Buckling Coefficient

The elastic plate buckling coefficients in Eq. (1) depend upon the aspect ratio of the plates and the boundary conditions (Timoshenko and Gere (1961)). Typical design equations use the lower-bound of these plate buckling coefficients for different loading and boundary conditions. For example, the lower-bound elastic buckling coefficient ($k_{flexure}$) for plates subjected to pure flexure is 23.9 for simply-supported (S-S) edge conditions and 39.6 for fixed (F-F) edge conditions. Consequently, it is important to ensure that the conclusions drawn in this paper with respect to the flange restraint on the web buckling strength represents the lower bound solution, and is independent of the unbraced length chosen in the FE simulations. For example, Fig. 13 shows the typical variation of $k_{flexure}$ with the web aspect ratio for a doubly-symmetric girder (D_c/D = 0.5) and a singly-symmetric girder (D_c/D = 0.625) with D = 2000 mm, $2D_c/t_w$ = 125, t_{fc}/t_w = 2.0. and D/b_f = 4 The buckling strengths of the web are plotted against the ratio of the total unstiffened length of the girder (L) to the clear depth of the web (D). The following is gleaned from Fig. 13.

1. It is observed that the variations in $k_{flexure}$ of I-girders fall within the limits of simply-supported and fixed-ended plates, although the maximum and minimum values occur at different aspect ratios for different girders.
2. Similar to plate buckling behaviour, the variation in $k_{flexure}$ decreases with increase in the plate aspect ratio. It is observed that $k_{flexure}$ is nearly constant for aspect ratios ≥ 2.0. Further studies are hence conducted on girders with aspect ratios of two to obtain the lower-bound solution.

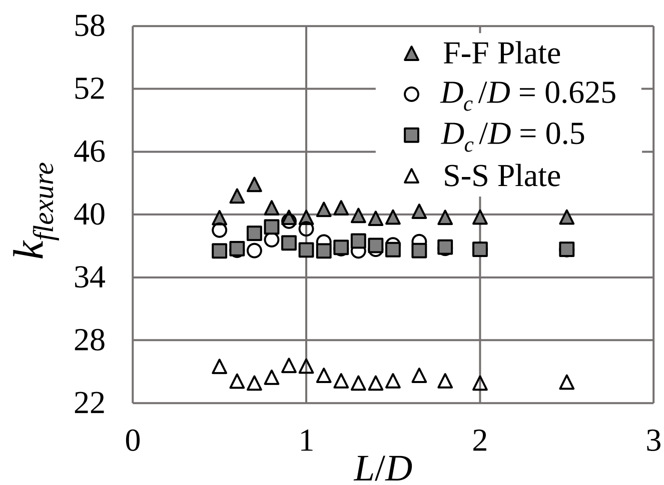

Figure 13: Bend-buckling behaviour of girder webs and plates for varying web aspect ratios (D/b_f = 4, $2D_c/t_w$ = 125 and t_f/t_w = 2.0)

4.3 Cross Sectional Parameters for Finite Element Simulations

The clear depth of the web in all girders is taken equal to 2000 mm. The other cross-sectional parameters are varied in the following ranges. A total of 191 girders are analyzed to study the influence of flanges on the web elastic flexural buckling capacity.

D/b_f = 4,5,6;

D_c/D = 0.40,0.50,0.65;

$2D_c/t_w$ = 70,100,150,200;

t_{fc}/t_w = 1.1 to 4.0

The compression flange slenderness ratios ($b_f/2t_{fc}$) are chosen to prevent flange local buckling. The range of $1/a_w$ studied in this paper is between 0.14 and 1.25, where, a_w is the ratio of two times the area of web in compression to the area of compression flange.

4.4 Influence of Flanges on the Web Elastic Flexural Buckling Coefficient

Fig. 14 shows the variation of $k_{flexure}$ with respect to $1/a_w$ in the 191 test girders. These values are calculated using Eq. (1), wherein the buckling strengths are obtained from the eigen value analyses in ABAQUS (2018). The results from the test simulations are compared with the elastic buckling coefficients that are calculated from the expression for non-compact web slenderness limit developed by Subramanian & White (2017). The following may be gleaned from Fig. 14.

1. The web buckling strength is clearly a function of the flange dimensions relative to the web dimensions, and vary from the lower bound for simply-supported plates to the upper-bound of fixed-ended plates.
2. The flange dimensions have little effect on the rotational restraint offered to the webs for $1/a_w > 0.51$. This is consistent with the observations made by Subramanian & White (2017) on the basis of full geometric and material non-linear analysis including the effects of lateral torsional buckling.

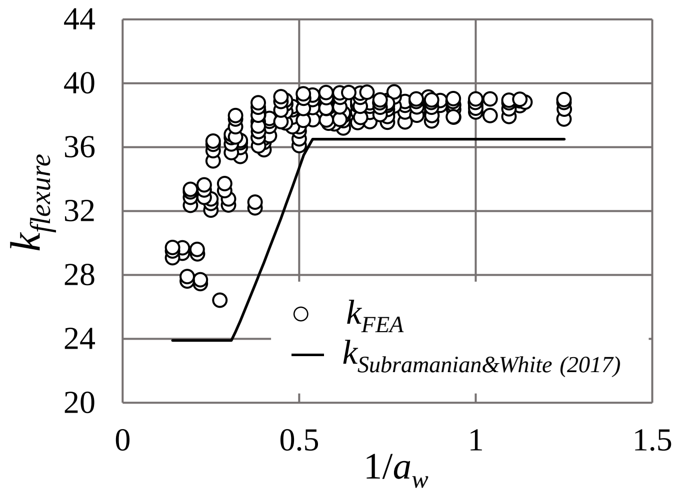

Figure 14: Comparison of computed web bend-buckling coefficient with Subramanian & White (2017)

It is clearly unconservative to assume a value of $k = 36$ for all girders (as per AISC (2016)), while it is also overly conservative to assume simply-supported edge conditions with $k = 23.9$ (as per Eurocode (CEN 2005)). These simplistic assumptions result in wrongful classification of slender webs (Class IV) as noncompact webs (Class III) and vice versa.

4.5 Behaviour of Doubly-Symmetric vs. Singly-Symmetric Girders

Frank and Helwig (1995) expressed the buckling coefficient of singly-symmetric girder webs (k_{usym}) as a function of D_c/D, and the elastic buckling coefficient of symmetrical girder webs (k_{sym}) with the same aspect ratios and boundary conditions as:

$$k_{usym} = \frac{k_{sym}}{4}\left(\frac{D}{D_c}\right)^2 \tag{2}$$

This is made possible by expressing the web slenderness for flexure of singly-symmetric I-girders as $2D_c/t_w$. The buckling coefficient proposed by the authors thus accounts for the girder asymmetricity, and the total web slenderness. The values of k_{usym} for a girder with D_c/D equal to

0.65 are 14.14 and 23.43 corresponding to simply-supported and fixed boundary conditions at the web edges respectively. The buckling coefficients obtained for girders with D_c/D equal to 0.65 ranges from 17.31 (for $1/a_w$ = 0.14) to 23.4 (for $1/a_w$ = 0.77). These values are obtained by substituting the buckling stress obtained from FE test simulations in the traditional plate buckling equation (Eq.(1)). Similarly, for a girder with D_c/D equal to 0.4, the value of k_{usym} corresponding to simply-supported and fixed boundary conditions at the web edges are 37.34 and 61.88 respectively. For a girder with D_c/D equal to 0.4, the minimum value of $1/a_w$ considered in the study is 0.73. The theoretical elastic flexural buckling coefficient (from Eq. (2)) corresponding to this girder is 59.69. The maximum value *of* $k_{flexure}$ obtained for a girder in this case is 61.97. These comparative values suggest that the buckling strengths obtained from the FE test simulations are consistent with the values obtained using the expression proposed by Frank and Helwig (1995). In this section, the influence of flange dimensions on web bend-buckling strengths for varying D_c/D ratios is presented.

4.5.1 Influence of Flange Dimensions on Web Bend-Buckling for Varying D_c/D Ratios

The buckling strengths obtained from the simulations are normalized with respect to the lower-bound buckling coefficient value corresponding to pure flexure loading and simply-supported boundary conditions ($k_{flexure,ss}$) i.e., 23.9. Fig. 15 distinguishes the normalized bend-buckling coefficients for D_c/D ratios of 0.65, 0.50 and 0.40. The detailed statistics are presented subsequently in Table 4. The following may be gleaned from Fig. 15 and Table 4.

1. For a given $1/a_w$, the values of $k_{FEA}/k_{flexure,ss}$ for doubly- and singly-symmetric girders with D_c/D equal to 0.65 are approximately equal. This indicates that the webs with D_c/D equal to 0.65 do not derive significant benefit from the thicker tension flanges, and that the critical flexural buckling stress is primarily a function of the compression flange. This is consistent with the observation of Timoshenko and Gere (1961) that the critical flexural buckling stress of a plate simply-supported on the compression side and fixed on the tension side is not different from a plate that is simply-supported on both the compression and tension sides.

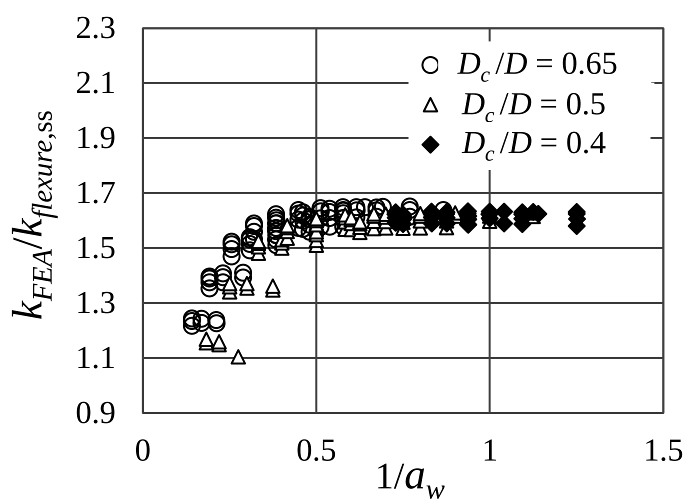

Figure 15: Influence of D_c/D on web bend-buckling coefficient for varying $1/a_w$ ratios

2. The study focuses on girders with the thickness of the compression or tension flanges atleast 1.1 times greater than the thickness of the web. Hence, the minimum value of $1/a_w$

for a girder with D_c/D equal to 0.4 is 0.73. The minimum value of $k_{FEA}/k_{flexure,ss}$ is 1.58 and a maximum value of 1.66. Thus, the webs have significant rotational restraint from the flanges in all these girders. As a result of this restraint from the compression flange, the mean value of $k_{FEA}/k_{flexure,ss}$ for girders with D_c/D equal to 0.4 is slightly larger (5%) than the mean for girders with D_c/D equal to 0.5 and D_c/D equal to 0.65. However, the maximum value of $k_{FEA}/k_{flexure,ss}$ in this case is equal to that obtained for doubly-symmetric girders. These values show that, for a girder with D_c/D equal to 0.4 conforming to the minimum flange-to-web thickness ratio prescribed in AASHTO (2016), the buckling coefficient is at least 1.58 times greater than the buckling coefficient corresponding to simply-supported boundary conditions (and nearly equal to fixed boundary conditions on a plate).

Table 4: Statistics for $k_{FEA}/k_{flexure,ss}$

Statistics	$k_{FEA}/k_{flexure,ss}$		
	D_c/D = 0.65	D_c/D = 0.50	D_c/D = 0.40
Mean	1.54	1.53	1.61
COV	0.08	0.08	0.01
Min	1.22	1.11	1.58
Max	1.65	1.63	1.63

4.6 Impact of True Boundary Condition at Web-Flange Junctions on Web Section Classification

This Section discusses the importance of estimating the true flexural buckling coefficient by discussing its impact on the section classification of the web subjected to flexure. The non-compact web slenderness limits for the girders (λ_{rw}) studied in this paper are calculated with the computed buckling coefficients from Eq. (1). These values are compared with those obtained from AISC (2016), the Eurocode (CEN 2005) and the improved non-compact web slenderness limit proposed by Subramanian & White (2017) in Figs. 16 and 17, respectively. The statistics for the comparison are listed in Table 5. The following points are observed from Figs. 16-17 and Table 5.

1. Fig. 16 shows that the web slenderness limit imposed by AISC (2016) is unconservative for 33 of the 191 girders (17% of total girders) considered in the study. The $1/a_w$ value of these girders ranges from 0.19 and 0.40. This is consistent with the observations made by Subramanian and White (2017). These 33 girders are unconservatively classified as noncompact webs (Class III webs), while they should be classified as slender webs (Class IV).
2. Fig. 16 shows that the web slenderness limit imposed by Eurocode (CEN 2005) is on average 36% conservative for all the girders considered. It can be noted from Table 5 that the assumption of simply-supported boundary conditions may be as conservative as 41%. This results in a conservative slender-web (Class IV) classification, while the webs, are essentially noncompact (Class III). This classification, if corrected shall result in a significant flexural strength increase throughout the beam design curve.
3. Fig. 17 shows that the noncompact web slenderness limit proposed by Subramanian & White (2017) provides a reasonable agreement with the results obtained from the FE test simulations. This classification is on average conservative by 8%, with a maximum conservative value of 24.7% for girders with small $1/a_w$ (less than 0.5). Subramanian & White conservatively limit $k_{flexure}$ to 36 instead of 39.6, consistent with the current AISC and

AASHTO provisions. Further, Subramanian et al. (2018) have shown that this equation provides the best correlation with experimental data and larger reliability indices as compared to the current AISC (2016) provisions.

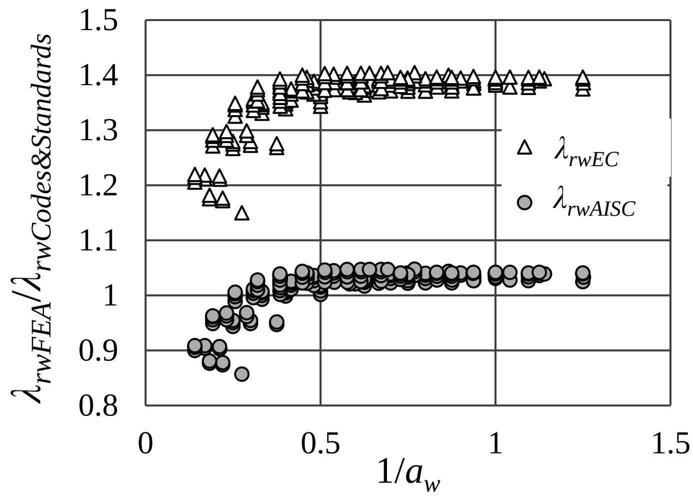

Figure 16: Comparison of non-compact web slenderness limit obtained using AISC (2016), Eurocode (CEN 2005) and true boundary conditions

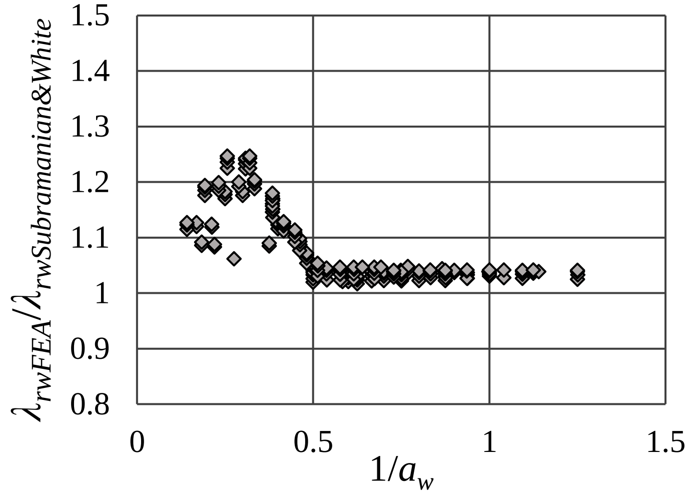

Figure 17 Comparison of non-compact web slenderness limit obtained using true boundary conditions and Subramanian & White (2017)

Table 5: Statistics for the comparison of non-compact slenderness limit obtained using test simulations with the provisions in international codes and literature

Statistic	$\lambda_{rwFEA}/\lambda_{rwAISC}$ AISC (2016)	$\lambda_{rwFEA}/\lambda_{rwEC}$ CEN (2005)	$\lambda_{rwFEA}/\lambda_{rwSubramanian\&White(2017)}$ Subramanian & White (2017)
Mean	1.02	1.36	1.08
COV	0.04	0.04	0.06
Min	0.86	1.15	1.02
Max	1.05	1.41	1.25

5. Conclusions

The paper presents findings on the influences of flanges on the web elastic buckling capacities when subjected to pure shear and uniform moment. In addition, the influence of transverse stiffeners on the elastic shear capacity is examined. Finally, the impact of the true flexural buckling capacities on the web section classifications is evaluated. The following list the notable conclusions from this work.

(a). Elastic shear buckling coefficient: It is observed that

1. The flange thickness relative to the web thickness has the most significant impact on the elastic shear buckling coefficient. The elastic shear buckling coefficient increases with an increase in the flange thickness upto a t_f/t_w equal to three.
2. Larger ratios of D/b_f (> 5) reduce k_v in I-girders with aspect ratios greater than three and may lead to unconservative design.
3. Webs with low slenderness values (< 150) require proportionally larger flanges to achieve higher elastic buckling capacities. This is especially true for web panel aspect ratios greater than two. The stocky webs when combined with smaller D/b_f (≤ 5.0) and smaller t_f/t_w (≤ 1.5) may develop elastic shear buckling strengths less than those of simply-supported plates.
4. The elastic shear buckling strengths of singly-symmetric I-girders are less than 6% of the strengths of doubly-symmetric I-girders, and may be neglected.
5. Transverse stiffeners significantly impact (< 25%) the elastic shear buckling strength in webs with panel aspect ratios of less than one.
6. While the design strength equations in Eurocode (CEN 2006) and AISC (2016) may be unconservative upto 60% for I-girders with web slenderness less than 150, they are highly conservative (≈ 60 - 70%) for I-girders with greater web slenderness ratios. The elastic shear buckling coefficient proposed by Lee et al. (1996) is unconservative by a maximum of 64% for panel aspect ratios more than two and slenderness ratios of web less than 150. It is also conservative by upto 30% for stiffened web panels with aspect ratio ≤ 1.0.

   The authors are in the process of developing equations for elastic shear buckling coefficients that account for both transverse stiffeners and flanges.

(b). Elastic web bend-buckling coefficient and web section classification: Eigen value buckling analyses are conducted for laterally braced steel I-girders subjected to uniform moment. The following conclusions are drawn on comparing the results with the various codes and available literature.

1. The bend-buckling strengths of I-section webs varies with the aspect ratios of the girders akin to a plate with ideal boundary conditions. This variation is negligible for aspect ratios greater than two.
2. The elastic buckling strengths of singly-symmetric I-girders subjected to flexural compression are typically greater than the doubly-symmetric girders with equal a_w, but the difference is not significant.
3. While the Eurocode (CEN 2005) gives a highly conservative non-compact web slenderness limit, AISC (2016) is unconservative for girders with small $1/a_w$. This results in an incorrect classification of slender webs as noncompact webs by AISC and vice versa by Eurocode.

4. The improved expression for non-compact web slenderness limit developed by Subramanian & White (2017) is a better estimate of the true web bend-buckling strengths. Hence, it is recommended to adopt the same for web classification for flexure.

Acknowledgement

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