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Unified stability design of steel structures using nonprismatic and curved members

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Abstract
This paper provides an overview of several research developments I have been privileged to be involved with during my 30+ years of engagement with colleagues within the Structural Stability Research Council, AISC, MBMA, AISI, FHWA, and AASHTO. Emphasis is given to key concepts and procedures captured within the recently-published AISC/MBMA Design Guide 25 Second Edition, *Frame Design Using Nonprismatic Members*, the applicability of these concepts and procedures to the design of arches, and related concepts and methods implemented in the AASHTO LRFD *Specifications* for the unified design of straight and horizontally curved bridge I-girders.

1. Introduction

I am indebted to so many people throughout my career, often via collaborations initiated within and strengthened by the Structural Stability Research Council (SSRC). My first formal exposure to the SSRC began in January 1983, when I was fortunate to take the graduate class *Structural Stability* with Teoman Peköz at Cornell. The 3rd Edition of the *Guide to Stability Design Criteria for Metal Structures*, the text *Principles of Structural Stability Theory* by Alexander Chajes, and the *Theory of Elastic Stability* monograph by Timoshenko and Gere were required purchases for the course. In addition, Dr. Peköz engaged us with assigned readings from Bleich's *Stability of Metal Structures* as well as reports from his research. As an undergraduate student at NC State, J.C. Smith had piqued my interest in plastic design (via Lynn Beedle's *Plastic Design of Steel Frames* text). Bill McGuire and John Abel were kind enough to pick me up as a fresh MS student in their research program on the inelastic design of steel frames using interactive computer graphics. I quickly developed a passion for research, spending long hours with articles and books, and writing structural analysis and computer graphics software. Bill introduced us to numerous papers in his two-semester course sequence on *Advanced Design and Behavior of Metal Structures*. Through Teoman, Bill and John, we were exposed to various articles by giants in the field such as Beedle, Johnston, Winter, Galambos, Yura, Chen, Trahair, Nethercot, and others.

I first attended the SSRC Annual Technical Meeting at the 4th International Colloquium on *Code Differences around the World*, held in New York City in April 1989. Other first-time attendees that year were Jerry Hajjar and Greg Deierlein. I was a fresh junior faculty member at Purdue University. Will Chen had proposed that SSRC form a new Task Group 29 on second-order

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inelastic analysis for frame design and had nominated me as the committee's chair. I recall feeling very intimidated before meeting with the SSRC Executive Committee, given the stature of the various committee members. However, Lynn Beedle was very gracious in welcoming my involvement in the Council. I learned later that he had raised questions about whether I understood the importance of experimental testing to validate computational methods. I believe Will and others convinced him that I did. I recall insightful presentations and engaging discussions on second-order inelastic analysis and design and its potential from Russell Bridge, Will Chen, Bill McGuire, David Nethercot, John Springfield, and Nick Trahair at an ad hoc session on the topic in New York City. Nick Trahair introduced us to the early Australian Standard (AS 4100) rules for Advanced Structural Analysis. These rules were a seminal model behind the present AISC *Specification* Appendix 1 for Design by Advanced Analysis. In addition, the AISC Direct Analysis Method of frame design has significant roots in the above discussions.

I met Ted Galambos at the meeting in New York City. During the first evening of the conference, several of us sat around chatting with Ted and others about the inelastic behavior of members and frames and other sundry topics. It was his 60th birthday. A couple of years ago, Ted celebrated his 90th birthday and I celebrated my 60th. I am very grateful for the early mentorship from Lynn, Will, Ted and others, and the many years of camaraderie with them.

There have been numerous other mentors and collaborators who have inspired me during my career, but I will not discuss those further at this point. A large number of key individuals are cited in the acknowledgments. I hope that this paper and the ensuing discussion at the 2022 Annual Stability Conference (ASC) will inspire others, even if in a small way, in their pursuits. Indeed, it is a fitting tribute to Bill McGuire's legacy that Ron Ziemian and I, having both conducted our doctoral studies under Bill's guidance, are giving our 2021 and 2022 Beedle Award presentations this year at the ASC. I hope that our presentations would make Bill proud.

In this paper, I would like to summarize a range of advancements in the stability design of girders, frames, and arches, many of which have grown out of the above efforts on second-order inelastic analysis for frame design. In addition, I would also like to highlight specific achievements in the design of horizontally curved I-girders that also connect to the above early developments.

2. Balancing Generality, Comprehensivity, and Simplicity in Nonprismatic Member Design

Have you ever needed to design a framing system involving stepped and/or web-tapered columns? Or a frame in which axial loads are introduced into members at intermediate positions along their length? Figures 1a to 1c show several structures having these characteristics. Or have you needed to design a variable web-depth plate girder with steps in the plate thicknesses and/or flange widths along its length (Fig. 1d)? The first edition of the AISC/MBMA Design Guide 25 (DG 25) (Kaehler et al. 2011) was published as an extension of the AISC 360-05 *Specification* to address these considerations. A key focus was on balancing generality, comprehensivity, and simplicity in tackling the corresponding design complexities. The presentations within the DG 25 first edition emphasized web-tapered members. The first edition also addressed the broader application of the subject methods to members containing cross-section transitions and/or axial loads applied at intermediate positions along their lengths, as well as the overall system design of frames using these types of members. However, this was accomplished in an abbreviated manner. The second edition of DG 25, *Frame Design Using Nonprismatic Members* (White et al. 2021), provides an expanded discussion of the various considerations associated with frame analysis and rules for member proportioning for these types of structures.

(a) Clear-span gabled portal frames with nonprismatic columns and roof girders (courtesy of Lee Shoemaker, MBMA)

(b) Crane support structure with gabled portal frames and stepped crane columns (from ellsenbridge-crane.com)

(c) Warehouse building with modular steel frames containing stepped crane columns (from finework-cranes.com)

(d) Highway bridge with variable web-depth I-girders (courtesy of Jason Provines, VDOT)

Figure 1: Example structures utilizing nonprismatic members

Aside from the handling of nonprismatic member geometry and nonuniform member axial load, a key focus of the DG 25 second edition is the characterization of two specific stability design attributes common to metal building frames:

(1) The influence of axial compression in rafters and roof girders, and

(2) The influence of leaning column P-Δ effects on the sidesway stability of modular frames (i.e., frames in which the roof girders or rafters are supported vertically by light interior columns, subdividing the frame into multiple bays).

The second edition of DG 25 introduces the following advancements pertaining to the design of frames containing general nonprismatic members:

- Calculation of column axial resistances using a streamlined, unified plate effective width procedure, extending the method in Section E7 of the AISC 360-16 *Specification* to nonprismatic members.
- Consideration of the substantial shear post-buckling strengths in thin unstiffened I-section webs, plus the contribution from inclined flanges, in member shear strength calculations, extending Section G2.1 of the *Specification* to nonprismatic members.
- Simplified estimation of member elastic lateral-torsional buckling (LTB) resistances, as well as refined elastic buckling predictions using thin-walled open-section beam calculations.
- Direct evaluation of general prismatic and nonprismatic column, beam, and beam-column design resistances using efficient inelastic buckling analysis procedures.
- Application of the most up-to-date recommendations for the AISC Direct Analysis, Effective Length, and First-Order Analysis methods of system stability design.

This new edition of DG 25 provides extensive examples illustrating the application of the recommended methods.

The following discussions offer a snapshot of some of the key concepts introduced by DG 25.

3. Unifying Concepts for Nonprismatic Member Design

For the calculation of member axial compressive resistance, the basic procedures discussed in DG 25 focus specifically on:

(1) The governing elastic buckling load (or stress) ratio

$$\gamma_e = \frac{P_e}{P_r} = \frac{F_e}{f_r} \tag{1}$$

which is a constant for a given member unbraced length, where $f_r = P_r / A_g$ at a given cross section, and

(2) The axial load or axial stress level, P_r or f_r, and the cross-section effective area, A_e, at a number of potentially critical cross sections along the unbraced length.

Any member subjected to axial compression has a buckling load ratio, γ_e, by which the required strengths (i.e., the internal stresses or forces from the applied loading) are multiplied to obtain the governing elastic buckling strength (i.e., $F_e = \gamma_e f_r$ or $P_e = \gamma_e P_r$). In general, $F_e = \gamma_e f_r$ and/or $P_e = \gamma_e P_r$ can be different at different cross sections along a member length. However, there is only one governing value of γ_e. Thus, the use of γ_e provides significant advantages for members with complex nonprismatic geometries, subjected to nonuniform or uniform axial compression. Furthermore, numerical buckling solutions provide γ_e directly as the eigenvalue, i.e., the multiple of the reference applied load, at incipient elastic buckling.

Given the value of $F_e = \gamma_e f_r$ at different locations along the length of a member subjected to nonconstant axial compression and/or containing a general nonprismatic geometry, the engineer can evaluate the nominal strength, P_n, at these locations. The P_n calculation is accomplished by applying Section E7 of the AISC 360-16 *Specification* at each of these locations. The calculation for the various potentially critical cross sections is akin to the evaluation of multiple members with distinct buckling characteristics in a general structure, each subjected to different axial compressive forces, P_r, using an overall buckling analysis of the structural system. Once the critical cross section (the one giving the largest demand-to-nominal strength ratio) is identified, the corresponding P_n calculation can be envisioned as being conducted on an equivalent uniformly-loaded prismatic member. This equivalent member has the same overall γ_e, and the same f_r / F_y and A_e / A_g [or f_r / (A_e / A_g) / $F_y = P_r / P_{ye}$], as the critical cross section (see Figure 2).

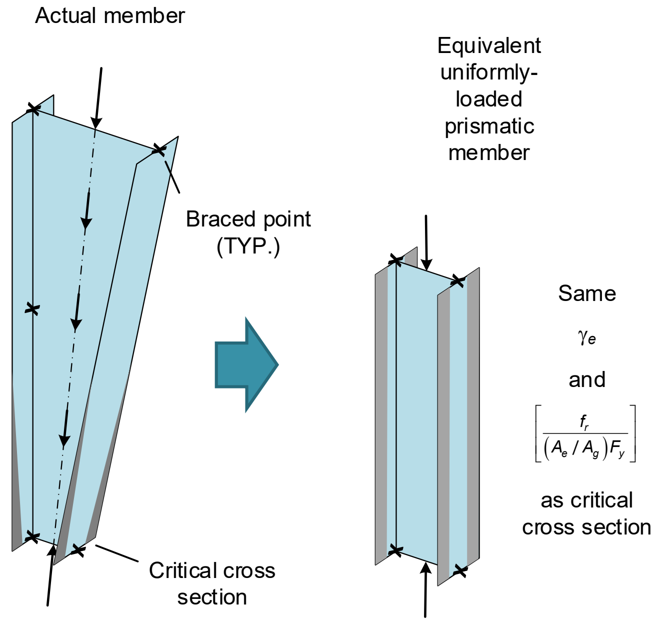

Figure 2: Equivalent uniformly loaded prismatic member concept

The above DG 25 approach is applicable with all three system stability analysis-and-design approaches in the AISC *Specification* – the Direct Analysis Method, the Effective Length Method, and the First-Order Analysis Method. In addition, a similar approach is recommended by DG 25 for calculation of the flexural resistance of I-section members. This approach captures the AISC Chapter F flange local buckling (FLB), tension flange yielding (TFY), and out-of-plane LTB limit states.

4. Effective Length Method, Direct Analysis Method, or First-Order Analysis Method?

4.1 Effective Length Method Considerations

Watwood (1985) touched on a particular anomaly of the Effective Length Method (ELM) for structural stability design. Members that have relatively small axial stress at incipient buckling of a frame tend to have large effective length factors, K, when considered as part of the evaluation of the overall structural system. In some cases, these K factors are justified, while in other cases, they are not. If the member participates significantly in the governing elastic flexural buckling mode, a large K value is justified. On the other hand, if the member is essentially undergoing rigid-body motion in the governing buckling mode, and/or if it predominantly serves to restrain flexural buckling of other members, a large K value is sometimes not justified. The distinction between these two situations requires significant engineering judgment. The K factor is only an index tied to the relation

$$P_{cr} = \frac{\pi^2 \tau EI}{(KL)^2} \tag{2}$$

in the case of a given prismatic member, where P_{cr} is the member axial force at the incipient buckling of the structural model considering just the axial forces in the model's members, no geometric imperfections, and no bending or pre-buckling displacements; τ is the column inelastic stiffness reduction factor (equal to 1.0 for elastic buckling); EI is the cross-section flexural rigidity of the member; and L is the reference unbraced length. The value of K is, in essence, always solved from Eq. 2 regardless of the method employed for its determination.

Effective length calculation procedures are typically implemented on subassemblies extracted from the overall structure. For instance, in multi-story buildings, K factors are commonly calculated on a story-by-story basis with limited consideration of the interaction between the stories. Suppose one conducts an elastic buckling analysis of an entire multi-story frame with a large number of stories. In that case, it is common to obtain K factors that are relatively large in the upper stories of the structure and relatively small in the lower stories. If a K factor is back-calculated for a typical girder of such a frame, the K value will be quite large since the axial force in the girder is relatively small.

DG 25 provides guidelines addressing these issues. Particular emphasis is placed on applying the ELM to typical metal building frames composed of nonprismatic members.

4.2 Advantages of the Direct Analysis Method

DG 25 explains that the AISC Direct Analysis Method (the DM) eliminates the above complexities by avoiding a focus on the stability limit states behavior of the structural system (or subassembly) in which all the members are subjected to pure axial compression (i.e., no bending). Frames are rarely subjected just to pure axial compression of their members. Instead, the DM focuses on the second-order load-deflection behavior of the geometrically imperfect structural system subjected to the actual required strength loading, rather than the bifurcation response at a higher load level associated with pure axial compression in the various members of the idealized geometrically-perfect structure. As a result, the DM provides significant advantages for design in that: (1) it may be used for all structures and load combinations, (2) it provides the most representative assessment of the actual internal forces and moments of the elastic analysis-and-design methods, and (3) it may be used to design frame members without calculation of K factors.

When using the DM, typically the in-plane flexural buckling strength of columns and beam-columns, P_{ni}, is calculated using the actual unbraced length with $K = 1.0$. However, this approach can also misrepresent the physical strength behavior in certain situations. For example, in clear-span portal building frames, the use of $K = 1.0$ to calculate the in-plane flexural buckling strength can be very conservative for the roof girders or rafters. Particularly in cases where the roof girder span is large and the eave height of the structure is relatively small, the columns can provide significant rotational restraint to the girder ends. Furthermore, the concept of a K factor is rather complex in itself when the roof girder has, for instance, multiple tapers and multiple steps along its length. DG 25 resolves this problem by the following extensions to the AISC *Specification*:

(a) For members with $\alpha P_r \leq 0.10 P_{eL}$ at all locations along their length, or stated more simply, for $\alpha/\gamma_{eL} \leq 0.10$ (where α, P_{eL}, and γ_{eL} are defined below), and where $A_{es} \geq 0.5A_g$, the member P_{ni} may be taken as the cross-section axial yield strength accounting for local buckling effects, $P_{ns} = A_{es}F_y$. This simplification is permissible because the in-plane stability effects are minor at the member level for columns and beam-columns that satisfy the above limits. The term P_{eL} here is the in-plane elastic flexural buckling load for the member unbraced length under consideration, assuming idealized simply-supported end conditions, and γ_{eL} is the corresponding elastic buckling load ratio. The term α is employed by the *Specification* to scale the required ASD "working" loads up to an ultimate strength design load level. It is equal to 1.0 for design by LRFD and 1.6 for ASD. Typical single-story metal building frame members often satisfy the $\alpha/\gamma_{eL} \leq 0.10$ and $A_{es} \geq 0.5A_g$ limits. (Note that A_{es} is the effective area associated with an axial stress equal to F_y; frame members practically always have $A_{es} \geq 0.5A_g$).

(b) If P-δ effects are included in the structural analysis model, and an appropriate out-of-straightness between the member ends is also included, P_{ni} may be taken as P_{ns} even when $\alpha/\gamma_{eL} > 0.10$. This is permissible because the combined reduced stiffness and out-of-straightness in the DM-based analysis account sufficiently for the in-plane stability effects at the member level. The appropriate member out-of-straightness is an imperfection of $0.001L$ in the direction that the member deforms (due to the applied loads) relative to a chord between its end supports or points of connection to other members, where L is the overall member unsupported length. A chorded representation of the out-of-straightness with maximum amplitude at the middle of the unsupported length is considered sufficient. For clear-span gabled frame rafters subjected to loads causing a net downward displacement at the ridge, this requirement may be implemented by shifting the ridge downward by $0.001L$, where L is the on-slope length between the columns. For an unusual situation where the loading may cause an upward movement of the ridge, the ridge should be shifted upward by $0.001L$.

The use of $P_{ni} = P_{ns}$ with the DM, based on satisfying one of the above two requirements, is the most accurate and the preferred approach for the in-plane stability design of rafters in clear-span frames. Within the above contexts, the load-deflection analysis of the DM sufficiently captures all the essential attributes of the in-plane stability behavior. Therefore, the member in-plane axial compressive resistance may be calculated as the axial compressive resistance of its cross sections. Furthermore, since the out-of-plane axial compressive resistance is always less than P_{ns} for any finite out-of-plane unbraced length, the out-of-plane buckling resistance always governs when the in-plane strength is taken as $P_{ni} = P_{ns}$.

The accurate design of rafters and roof girders using the ELM requires the recognition of end restraint from the columns within an elastic buckling analysis or the related $K < 1.0$ solution. Unfortunately, the lowest eigenvalue buckling mode may correspond to $K > 1.0$ in these members. As stated previously, the use of $K = 1.0$ with the DM for calculating P_{ni} in roof girders or rafters of clear-span portal frames can result in a significantly conservative characterization of the axial compressive resistance in these members. The most effective way out of this quandary of in-plane stability design assessments is to focus on estimating the second-order load-deflection response due to the actual strength design loads applied to the structure on the demand side, and to focus on the cross-section-based resistance $P_{ni} = P_{ns}$ on the resistance side.

4.3 Limitations of the First-Order Analysis Method

Regarding the First-Order Analysis Method (FOM), DG 25 recommends that for frames in which the internal axial force in any of the girders or rafters exceeds $0.08P_{eL}$ (i.e., when $\alpha/\gamma_{eL} > 0.08$), the FOM should be limited only to preliminary design. In this context, P_{eL} is the nominal in-plane elastic flexural buckling strength of the girder or rafter, based on the on-slope length between the columns and assuming simply-supported end conditions. The simplifying approximations embedded in the FOM can become suspect for frames that fall outside of these limits.

5. Advanced Calculations

Generally, the basic procedures recommended in DG 25 require the calculation of member elastic buckling load ratios, γ_e, followed by a mapping to the corresponding design resistances. However, suppose the stiffness reduction factors (SRFs) associated with the *Specification* strength curves are embedded within the buckling calculations. In that case, the buckling analysis can be configured to provide the column, beam, or beam-column design resistances directly. This type of inelastic buckling analysis is discussed in the DG 25 second edition as a supplement to the basic or more routine methods. The advantage of inelastic buckling analysis is that it can more rigorously account for a wide range of attributes such as:

- Nonprismatic geometry,
- Moment gradient,
- Variations in axial force along member lengths,
- Load height,
- Member end restraint,
- Member continuity effects across braced locations, and
- Beam-column strength interactions.

In addition, inelastic buckling analysis removes the need for tedious and relatively inaccurate C_b, K, and beam-column strength interaction calculations.

It should be noted that all of the above methods are focused on the design of planar structural systems in which the members are loaded predominantly within the plane of the structure (although the member strength may be governed by in-plane or out-of-plane, three-dimensional, limit states). The in-plane actions and strength limits are addressed most effectively by focusing on the in-plane second-order load-deflection response. In contrast, the out-of-plane strength assessments are handled most effectively by calculations that are rooted (implicitly or explicitly) in an eigenvalue buckling analysis (elastic or inelastic). Suppose the DM approach of focusing on the second-order load-deflection response of the geometrically imperfect structure is employed in the out-of-plane direction. In that case, the planar frame solution becomes a more complex three-dimensional load-deflection analysis problem. Handling of these complexities by a theoretical out-of-plane buckling (bifurcation) analysis simplifies the out-of-plane calculations substantially. Figure 3 shows a representative inelastic out-of-plane buckling mode of a tapered member from DG 25. The darker arrows with slashes denote constrained displacements and rotations within the model. The lighter non-slashed arrows labeled *M* and *P* indicate the applied axial force and moment at the right-hand end of the member. Figure 4 illustrates the variation of an inelastic stiffness reduction factor (*SRF*) applied to the rigidities EI_y, EC_w, and GJ along the member length. Details of the calculations are explained in the DG 25 second edition and by White et al. (2016 and 2021).

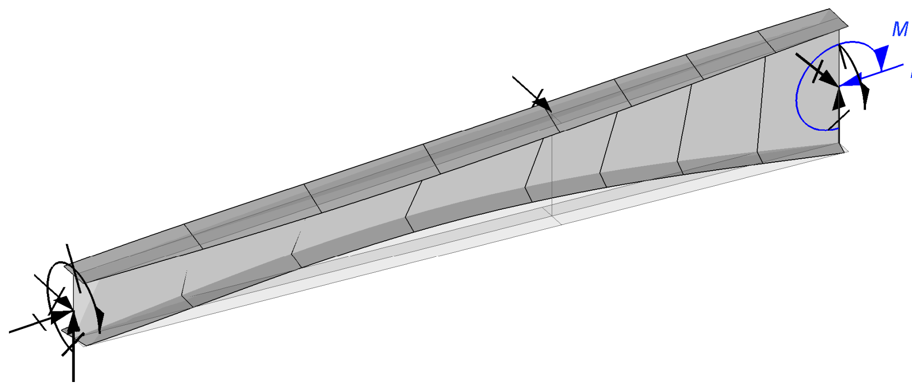

Figure 3: Inelastic out-of-plane buckling mode of a tapered member subjected to axial compression and bending, from AISC/MBMA Design Guide 25 (White et al. 2021)

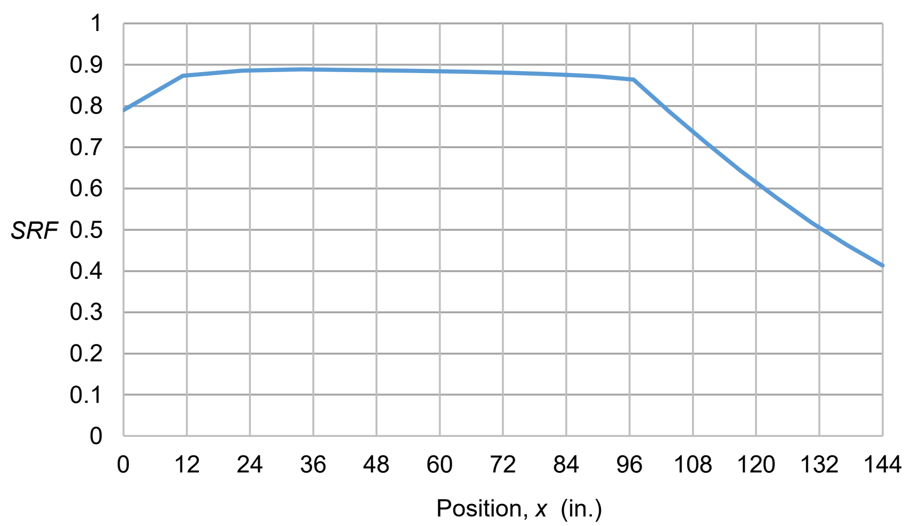

Figure 4: Variation of the inelastic stiffness reduction factor (*SRF*) along the member length at incipient inelastic out-of-plane buckling of the member shown in Fig. 3

All of the above methods stop short of the highest level of rigor of structural analysis for member and structural system design. In the lexicon of the AISC *Specification*, the term Advanced Analysis is reserved for the highest level of analysis. In a few words, Advanced Analysis commonly refers to a method of analysis in which, once the analysis has been conducted up to or beyond the required strength load levels, the engineer can conclude that the structure is sufficient to resist the required loads. No further member resistance calculations are necessary. That is, the structural analysis is fully capable of capturing the strength limits without the separate application of any member resistance calculations. Unfortunately, a complicating factor is that the definition of what constitutes an Advanced Analysis can vary depending on the nature of the structural system and its members and other components. This is where some of the distinctions and definitions of "advanced" become cloudy and confusing.

In the AISC *Specification*, Advanced Analysis implies a comprehensive three-dimensional load-deflection analysis of a structure or structural subassembly. However, the satisfaction of comprehensivity depends on which strength limit states significantly impact the response for a given problem. For instance, if local buckling has a significant contribution to the strength limit states, local buckling effects must be addressed rigorously in some manner within the Advanced Analysis calculations. On the other hand, if the member cross-sections are sufficiently stocky, then local buckling effects are insignificant. In this case, the member cross-sections must be sufficiently stocky such that local buckling does not significantly impact the ductility required for inelastic redistribution of the local demands. Tools such as the continuous strength method (Gardner 2008) provide a refined means for assessing the local demand-to-capacity, focusing on the strain or deformation demands. Alternatively, traditional plastic design cross-section b/t limits can be checked. If the local buckling response is not sufficiently limited (by limiting b/t), a more sophisticated advanced analysis model is necessary to capture the local buckling effects.

In many situations, the unbraced lengths of I-section members are not small enough to discount the influence of out-of-plane LTB limit states on the response. Proper assessment of the LTB response of open-section members requires the consideration of warping (e.g., cross-bending of the flanges of an I-section member) based on thin-walled open-section beam theory at a minimum. Furthermore, it is well established that the onset of yielding, including residual stress effects, can significantly impact the LTB resistance. The most rigorous way to address these considerations is via a distributed plasticity analysis capable of capturing the spread of yielding through the cross section and along the length of the structural members, including the modeling of residual stresses. In addition, the out-of-plane sweep of the compression flange(s) generally has a measurable influence on the LTB response. Therefore, appropriate out-of-plane sweep imperfections also must be modeled in a rigorous Advanced Analysis solution. Manual definition of the appropriate geometric imperfections and residual stresses is not practical if these types of solutions are to become more routine. Therefore, although the technology has great potential, it is my opinion that it will be necessary for software to handle these definitions in an automated manner (with control by the engineer) for Advanced Analysis to achieve the most useful application in structural engineering practice.

Given the above challenges, the techniques described previously (i.e., the use of elastic buckling load ratios, γ_e, to generalize *Specification* rules, and/or the use of inelastic buckling analysis methods) were the best choice for practical consideration of out-of-plane strength limit states in DG 25 at the present time. DG 25 predominantly targets the design of planar members and frames loaded within the corresponding plane, but in which the strength limit states may be (and often are) associated with an out-of-plane failure. The in-plane stability problem is handled well, on the demand side, by the DM's load-deflection-based procedures. The out-of-plane stability problem is handled best on the resistance side, via buckling (bifurcation) analysis approximations, or by using rigorous theoretical elastic or inelastic eigenvalue buckling calculations where desired.

It is important to understand that the internal required force predictions from the DM are not "perfect." They typically provide a reasonable match with Advanced Analysis solutions, but there will be some differences. The same can be said for any structural analysis based on the assumption of linear elastic material behavior (with reduced or nominal stiffness). I like to refer to the DM analysis solution as a "poor-folks" version of a distributed plasticity test simulation. The objective is to obtain a reasonable coarse representation of the response of the physical structure. When considering the predictions of the internal force demands, I believe it is valuable to remember the lessons from the moment balancing method discussed in Lynn Beedle's *Plastic Design of Steel Frames* text. There is no unique required internal force distribution for a given design. One need only determine a statically admissible set of internal forces that does not exceed the available resistance anywhere in the structure, assuming adequate ductility. The consideration of second-order effects complicates this deliberation a bit; however, the concept is still the same but applied in the structure's deformed configuration. I recall Ted Galambos' words of wisdom that structural engineers need to understand plastic design so that they will be "less afraid of their structures." Even if the DM analysis were a perfect match to Advanced Analysis, there is always some sensitivity of the Advanced Analysis results to the details of the residual stresses, initial geometric imperfections, applied load positions, etc. This is not to say that the approximations by the DM analysis rules cannot be improved. However, "perfection" is an unachievable goal.

6. Stability Design of Arches

The concepts and procedures from DG 25 also are relevant to the structural design of arches. Figure 5 shows an elegant tied-arch bridge designed using the Direct Analysis Method (Nair et al. 2016; 2019). For the in-plane stability assessment of this type of structure via the DM, one considers an out-of-straightness within the plane of the arch rib that is affine to the buckling mode of the arch. The in-plane buckling mode of an arch rib is an S shape. Once the out-of-straightness is included in the analysis, the axial resistance of the arch rib may be taken as its cross-sectional resistance (P_{ns}, based on $L = 0$) at any location along its length. In addition, the prior discussions pertaining to the handling of nonprismatic geometry are applicable for the design of arches in cases where the cross-sectional geometry varies along the length.

Fig. 5: Tied arch bridge designed using the Direct Analysis Method (courtesy of Shankar Nair, EXP Design)

In his discussions, Nair (2016) emphasizes that DM principles were instrumental in justifying the lightweight Vierendeel bracing system between the arch ribs in the bridge shown in Fig. 5. In fact, for tied-arch bridges, the out-of-plane stability effects tend to be more significant than the in-plane stability effects. In the plane of the arch, the second-order effects in the rib and tie counteract one another. The horizontal components of the rib compression and the tie tension are equal from fundamental statics. Also, the vertical displacements of the rib and the tie are approximately equal, neglecting the small hanger elongation. Therefore, the destabilizing effect of the rib vertical displacements is balanced by the restoring effect of the tie displacements. Nair (1986) shows a simple example demonstrating that tied arches do have a finite in-plane buckling load; however, the in-plane structural system stability is greatly enhanced by the tension tie. Arches without a tension tie can have substantially larger in-plane second-order effects. Either with or without the tie, the in-plane second-order amplification is most significant for load combinations involving unbalanced loadings, producing structural system displacements that are more affine to the structure's fundamental buckling mode.

The DM may be applied with relative ease to consider both the in-plane and out-of-plane stability effects in the structure shown in Fig. 5 since the arch ribs are closed box sections with substantial torsional stiffness. For the out-of-plane stability assessment, the essential geometric imperfection is a potential overall sweep of the arch ribs in the out-of-plane direction in the shape of a half sine wave or parabola. This overall sweep can be modeled easily by an equivalent (i.e., "notional") uniformly distributed out-of-plane lateral load and corresponding equilibrating end shears at the arch supports.

According to Dr. Nair, the design analysis of the bridge in Fig. 5 entailed a three-dimensional model with 4695 nodes, 1314 frame elements, and 4444 shell elements (shell elements were used to model the regions at the intersections of the arch ribs and ties). Furthermore, all the calculations were second-order, requiring 488 separate structural analyses for the various load combinations and effects considered for the design. Nair (2016) points out that in a similar tied-arch bridge he designed earlier in his career, the in-plane second-order effects were on the order of 7 %, while the out-of-plane second-order effects were five times larger.

In all situations, the essential concept for the in-plane and out-of-plane analysis for the DM is the following: Calculate a reasonable estimate of the *actual* second-order axial forces and moments within the structure, considering the influence of the internal forces acting through the structural displacements plus plausible initial geometric imperfections. This concept can be applied readily for general design, including the design of other arch structures such as network tied-arch bridges, arch bridges with inclined (non-vertical) ribs, etc.

One additional consideration that can be important to the proper stability design of some arches is the assessment of out-of-plane stability accounting for the influence of the vertical curvature. For unbraced lengths in vertically curved members such as arch ribs, the vertical curvature reduces the LTB resistance when the unbraced length is subjected to moments causing compression in the flange farthest from the center of curvature (that is, moments that "straighten" the arch rib). Conversely, the LTB resistance is increased by moments causing compression on the flange closest to the center of curvature; that is, moments that increase the curvature of the rib. It is safe and sufficient to neglect the increases in the LTB resistance due to these effects. For typical unbraced lengths of arch ribs with L_{db}/R greater than 0.20 (where L_{db} is the developed length between the brace points and R is the minimum radius of curvature of the arch rib), subject to moments that would reduce the LTB resistance, 0.90 is a reasonable lower bound on the reduction in the elastic LTB resistance. This reduction may be applied conservatively to the C_b modifier (AASHTO 2020). Of course, arch ribs in bridges are typically closed box sections, and therefore they have ample LTB resistance despite this reduction. The adjustment to the C_b modifier for arch ribs composed of doubly symmetric open or closed sections, with large unbraced lengths between the out-of-plane bracing members, may be determined more rigorously by a set of closed-form equations provided by Dowswell (2018). For arch ribs composed of singly-symmetric open or closed sections, the adjustment to the C_b modifier may be determined by solving equations provided by Trahair and Papangelis (1987).

7. Behavior and Design of Horizontally Curved I-Girders

Similar to DG 25, and to the routine handling of LTB in Section F of the AISC *Specification*, the AASHTO (2020) LRFD Bridge *Specifications* address the flexural resistance of straight and curved bridge girders using a buckling analysis-based approach. In fact, the AASHTO LRFD *Specifications* apply a buckling analysis-based (i.e., ELM-based) approach that in effect treats the girder flanges as effective beam-columns. The flange lateral bending is the bending moment, and the axial stresses due to major-axis bending correspond to the axial load in these effective beam-columns. The flange lateral bending may be due to lateral loads, such as wind, as well as warping (i.e., flange cross-bending) from the torsion due to horizontal curvature or eccentric loads from overhang brackets during construction, etc. Given that bridge design necessitates the detailed consideration of moving live loads, routine design of bridge I-girders is best suited to this type of approach (versus the use of a second-order load-deflection approach, such as the application of the DM for in-plane analysis and design of metal buildings, or the in-plane or out-of-plane design of the arch ribs in a case such as the bridge discussed in Section 6).

7.1 The AASHTO LRFD One-Third Rule

The basic form of the AASHTO (2020) resistance equations that account for the combined effects from major-axis bending and flange lateral bending is

$$f_{bu} + \frac{1}{3} f_\ell \le \phi_f F_n \tag{3}$$

in the AASHTO (2020) Section 6, where the major-axis bending resistance is expressed in terms of flange stress, and

$$M_u + \frac{1}{3} f_\ell S_x \le \phi_f M_n \tag{4}$$

in the AASHTO (2020) Appendix A6, where the major-axis bending resistance is expressed in terms of bending moment. The variables in these equations are as follows:

- f_{bu} = the elastically-computed flange major-axis bending stress,
- f_ℓ = the elastically-computed second-order flange lateral bending stress,
- $\phi_f F_n$ = the factored flexural resistance in terms of the flange major-axis bending stress,
- M_u = the member major-axis bending moment,
- S_x = the elastic section modulus about the major-axis of the cross section to the flange under consideration, and
- $\phi_f M_n$ = the factored flexural resistance in terms of the member major-axis bending moment.

Equations 3 and 4 are referred to by AASHTO (2020) as the one-third rule. These equations are simple, yet they do an excellent job of characterizing the various strength limit states that can govern the resistance of I-girders in skewed and/or horizontally curved bridges.

AASHTO (2020) targets Eq. 3 to assess the strength of slender-web noncomposite members, slender-web composite members in negative bending, and noncompact composite members in positive bending. In the limit that the flange lateral bending stress f_ℓ is zero, Eq. 3 reduces to the basic member check $f_{bu} \leq \phi_f F_n$ for major-axis bending alone. In the AASHTO Section 6, the flange yield strength, F_{yf}, is the maximum potential value of F_n. However, F_n can be less than F_{yf} due to slender-web bend buckling and/or hybrid-web yielding, lateral-torsional (LTB), or compression flange local buckling (FLB) effects.

AASHTO (2020) provides Eq. 4 for checking the strength limit states of straight noncomposite members or composite members in negative bending that have compact or noncompact webs, and for checking compact composite members in positive bending. For these member types, $\phi_f M_n$ can be as large as $\phi_f M_p$, where M_p is the section plastic moment resistance. However, Eq. 3 may be used as a simple conservative resistance check for all types of I-section members. AASHTO (2020) Article 6.10 emphasizes this fact by relegating the use of Eq. 4 to its Appendix A6.

In the application of Eqs. 3 and 4, the stresses f_ℓ and f_{bu}, and the moment M_u, are taken as the largest values throughout the unbraced length when checking against the base flexural resistance $\phi_f F_n$ or $\phi_f M_n$ associated with LTB. This is consistent with the proper handling of the moment when applying the AASHTO and AISC interaction equations for a general beam-column subjected to combined axial load and bending. The stress f_{bu} in Eq. 3 and the moment M_u in Eq. 4 are analogous to the axial load in a general beam-column, and the stress f_ℓ is analogous to the beam-column bending moment. The moment M_u is analogous to axial loading since it produces axial stresses in the flanges. When checking FLB or TFY, f_ℓ, f_{bu}, and M_u may be determined as the corresponding values at the cross section under consideration. Generally, Eq. 3 or 4, as applicable, must be checked for each flange, and both the FLB and LTB resistances must be checked for the compression flange in determining F_{nc} or M_{nc}. The check providing the largest ratio of the left-hand side to the right-hand side of these equations governs.

Equations 3 and 4 are valid for all types of I-section members that satisfy the limits

$$L_b/R \leq 0.1 \tag{5}$$

where L_b is the unsupported length between the cross-frame locations and R is the horizontal radius of curvature,

$$L_b \leq L_r \tag{6}$$

where L_r is the unbraced length limit beyond which the base LTB limit state is elastic, and

$$f_\ell \leq 0.6\, F_{yf} \tag{7}$$

The first of these limits is a practical upper bound for the subtended angle between the cross-frame locations (for constant R). However, Eqs. 3 and 4 have been observed to perform adequately in cases with L_b/R larger than 0.2 (White et al. 2001). Equation 6 is a practical upper bound for the unbraced length L_b beyond which the second-order amplification of the flange lateral bending stresses can be particularly large. The rationale for Eq. 7 is discussed below.

7.2 Calculation of Flange Lateral Bending Stresses

Various methods may be used for calculating the flange second-order elastic lateral bending stresses f_ℓ. AASHTO (2020) gives simple equations for estimating the first-order lateral bending stresses, $f_{\ell 1}$, due to the torsion associated with horizontal curvature, the torsion from eccentric concrete deck overhang loads acting on cantilever forming brackets placed along exterior girders, and the flange lateral bending due to wind load. However, it is becoming more and more common to calculate $f_{\ell 1}$ from 3D FEA bridge models for complex bridge geometries. In these solutions, the girder flanges are typically modeled using frame elements and the webs are modeled using shell elements.

Similar to the amplification of internal bending moments in beam-column members, flange lateral bending stresses are amplified due to stability effects. However, in routine girder bridge design, it is impractical to calculate second-order stresses associated with the moving live loads via a general-purpose second-order analysis. Therefore, when Eq. 3 is applied for checking the compression flange, AASHTO (2020) provides the following simple lateral bending amplification equation to approximate the second-order effects:

$$f_\ell = \left(\frac{0.85}{1 - \dfrac{f_{bu}}{F_{cr}}} \right) f_{\ell 1} \geq f_{\ell 1} \tag{8}$$

In this equation:

F_{cr} = the compression flange elastic LTB resistance for compact- or noncompact-web members or the elastic LTB resistance times the web load-shedding factor R_b for slender-web members,

$f_{\ell 1}$ = the first-order compression flange lateral bending stress at the section under consideration (for checking of FLB), or the largest first-order compression flange lateral bending stress within the unbraced length (for checking of LTB), and

f_{bu} = the largest value of the compression flange major-axis bending stress within the unbraced length under consideration.

A similar equation, but in terms of the moments, is employed with Eq. 4. Amplification of the tension flange lateral bending stresses is not necessary since the second-order effects tend to be relatively minor in flanges subjected to tension. White et al. (2001) show that Eq. 8 gives an accurate to conservative estimate of the compression flange second-order lateral bending stresses.

When determining the amplification of $f_{\ell 1}$ in horizontally curved I-girders with $L_b/R \geq 0.05$, AASHTO (2020) Article C6.10.1.6 recommends that F_{cr} in Eq. 8 may be calculated using $KL_b = 0.5L_b$. The use of $KL_b = 0.5L_b$ for girders with $L_b/R \geq 0.05$ gives a better estimate of the amplification of the bending deformations in the unbraced lengths within the spans, where the boundary conditions at the brace points are approximately symmetrical. The use of $KL_b = 0.5L_b$ is based on the observation that an unwinding stability failure of the compression flange is unlikely for magnitudes of the horizontal curvature larger than 0.05.

The basic amplification factor in Eq. 8 is a practical option for loading cases involving vehicular live load. However, in cases where the amplification of construction stresses is significant, a viable alternative is to conduct an explicit geometric nonlinear (second-order load-deflection) analysis to determine the second-order effects within the superstructure more accurately.

7.3 One-third Rule Concept

Figure 6 compares Eq. 4 to the theoretical fully plastic resistance for several noncomposite doubly symmetric cross sections with compact flanges and compact webs. Figure 7 shows a typical fully plastic stress distribution on this type of cross section. The equations for the fully plastic cross-section resistances are based on the original research by Mozer et al. (1971) and are summarized by White and Grubb (2005). The specific stress distribution shown in Fig. 7 is associated with equal and opposite lateral bending in each of the equal-size flanges (i.e., warping of the flanges due to nonuniform torsion). However, the solution is the same if one considers equal flange lateral bending moments due to minor-axis bending.

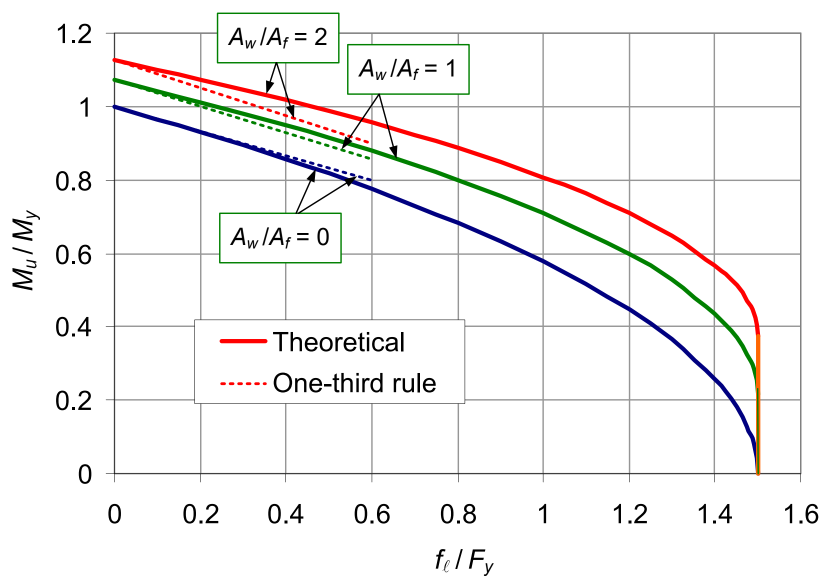

Fig. 6: Comparison of the AASHTO (2020) one-third rule equation to the theoretical fully-plastic cross-section resistance for several doubly symmetric noncomposite compact-flange, compact-web I-sections (adapted from White and Grubb (2005))

One can observe that, within the limit given by Eq. 7, the one-third rule equations (Eqs. 3 and 4) provide an accurate to somewhat conservative estimate of the theoretical cross-section resistances for the different web-to-flange area ratios, A_w/A_f, shown in Fig. 6. Moreover, in the limit that A_w/A_f is taken equal to zero, the same approximation is provided by both Eqs. 3 and 4.

The comparison of the theoretical and approximate equations shown in Fig. 6 is helpful in gaining a conceptual understanding of the one-third rule equations in the limit of compact-flange, compact-web, compactly-braced noncomposite members. Schilling (1996) and Yoo and Davidson (1997) have presented other useful cross-section yield interaction relationships applicable to these cases. However, cross-section yield interaction equations are limited in their ability to fully characterize the combined influence of distributed yielding along the member lengths along with the various stability effects (FLB, LTB, and web bend buckling). For instance, yield interaction equations generally do not reduce to the resistance equations for straight members subjected to major-axis bending in the limit that $f_\ell = 0$.

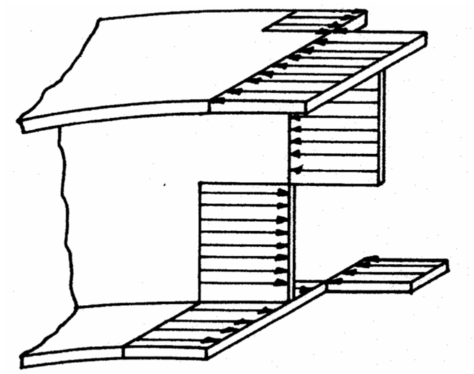

Fig. 7: Sketch of fully plastic stress distribution, including flange lateral bending

Equations 3 and 4 are a basic extension of the above one-third rule approximations of the theoretical cross-section yielding resistances to address the influence of stability limit states. This extension is accomplished simply by changing the flange yield strength, F_{yf}, to $\phi_f F_n$ in Eq. 3 and by changing the section plastic moment resistance, M_p, to $\phi_f M_n$ in Eq. 4. The 1/3 coefficient accurately captures the strength interaction including the various yielding and stability effects (White et al. 2001). The extension from cross-section yield interaction equations to the member strength equations is ad hoc. However, it is similar in many respects to the development of the AISC and AASHTO general beam-column interaction relationships. The shape of the interaction (i.e., the slope of the line relating f_{bu} and f_ℓ in Eq. 3 or M_u and f_ℓ in Eq. 4) is obtained from curve fitting. Equations 3 and 4 are thus semi-analytical and semi-empirical. White and Grubb (2005) summarize the correlation of these equations with analytical, numerical, and experimental results. These developments are tied to the early activities discussed in Section 1; advanced finite element simulations of experimental tests were employed extensively in the referenced research studies. These simulation studies were combined with targeted physical testing.

Dowswell (2018) discusses an approach similar to the above AASHTO one-third rule, but utilizing a form of the AISC 360 beam-column interaction Eq. H1-1a. This equation can be applied to cases with flange lateral bending stress larger than the limit specified by Eq. 7, such as members subjected to large minor-axis bending combined with major-axis bending. However, within the limits of $L_b/R \leq 0.2$, $L_b \leq L_r$ (Eq. 6) and $f_{bu} \leq 0.6F_{yf}$ (Eq. 7), Eqs. 3 and 4 provide a more accurate, less conservative characterization of the strength of I-section members subjected to major-axis bending combined with flange lateral bending from any source.

8. Unfinished Business

There are numerous areas of potential further improvement in girder, frame, and arch stability design methods. I would like to offer just a few recommendations beyond the ones alluded to in the previous discussions. These recommendations are referenced to the AISC *Specification*; however, comparable considerations apply to the AASHTO LRFD *Specifications*:

(1) There are various exclusion clauses allowing for simplified application of the Direct Analysis Method (the DM) in the AISC *Specification*. For instance, the *Specification* permits structures where the second-order amplification of the sway displacements is smaller than 1.7 (based on the reduced stiffness employed by the DM) to be analyzed for lateral load combinations neglecting out-of-plumbness effects. In these situations, out-of-plumbness effects tend to be negligible compared to the lateral load effects. However, for gravity-only load combinations, the modeling of out-of-plumbness (or the equivalent notional lateral loads) is always required. It should be possible to define a limit on the second-order amplification under the gravity load, or a γ_{cr} value corresponding to incipient elastic buckling of the structural system, at which the geometric imperfections may be neglected for the DM structural analysis also for the gravity-only load combinations. This would eliminate the annoyance of having to model geometric imperfection effects when they may be negligible for certain classes of structures.

(2) The AISC (2016) and (2022) Section F4 and F5 Tension Flange Yielding (TFY) limit state for singly-symmetric I-section members with a larger flange in compression, and hence $S_{xc} > S_{xt}$, is a somewhat artificial limit that amounts to disallowing any nominal yielding of the tension flange for slender-web sections. FEA simulation and experimental studies (Slein et al. 2021) show substantial capacity for inelastic redistribution of the tension flange stresses in these types of members. Simple calculation methods have been developed that employ only two flexural resistance limit state checks, compression flange local buckling (FLB) and lateral-torsional buckling (LTB), eliminating any explicit TFY check. These limit state calculations use the *actual* yield moment to the compression flange (accounting for the early yielding in flexural tension). This approach can dramatically shorten the AISC Chapter F provisions while also providing larger and more accurate calculated strengths.

(3) The AISC (2016) and (2022) Chapter F FLB provisions do not recognize the beneficial gains in strength due to the flange stable post-local buckling response. The downside of recognizing these gains in design calculations is the possible need to consider significant local-global buckling interaction. Latif and White (2022) have recommended a simplified approach that recognizes these benefits via a minor modification of the current AISC calculation procedures. These calculations avoid the need to consider the local-global buckling interaction problem explicitly.

(4) Subramanian et al. (2018) have demonstrated that the reliability index associated with the AISC (2016) and (2022) LTB strength calculations for built-up I-section members is significantly smaller than the target value of 2.6 for unbraced lengths in the vicinity of $KL_b = L_r$. Slein et al. (2021) have confirmed this assessment experimentally and by FEA test simulation. The fundamental issue resides in the onset of yielding that occurs within unbraced lengths due to the amplified flange lateral bending stemming from unavoidable flange sweep imperfections. Slein et al. (2021) have shown this problem can be addressed acceptably by a slight reduction in the nominal stress, F_L, at which inelastic LTB effects are deemed to be significant.

(5) Liang et al. (2021) have demonstrated that thin-web I-section members with relatively large C_b values can experience a significant reduction in their elastic LTB resistance due to web distortion effects exacerbated by web shear stresses. Deshpande et al. (2021) have

demonstrated that these effects can combine with the onset of compression flange yielding at flange stresses in the vicinity of and larger than F_L. As such, the flexural resistances predicted by the AISC *Specification* can be impacted substantially. Potential solutions are intrinsic in the calculation and/or application of the C_b factor.

9. Closing Remarks

This paper has presented an overview of several research developments I have been privileged to be involved with during my career. Emphasis has been given to key concepts and procedures captured in the recently-published AISC/MBMA Design Guide 25 Second Edition, the applicability of these concepts and procedures to the design of arches, and related concepts and methods implemented in the AASHTO LRFD *Specifications* for the unified design of straight and horizontally curved bridge I-girders. Cases have been highlighted where member and system stability design solutions are best conducted using a second-order load-deflection analysis of the geometrically imperfect structure, including nominal stiffness reduction effects (i.e., the Direct Analysis Method approach). In addition, cases have been highlighted where the design can be accomplished most effectively using a buckling (bifurcation) analysis approach.

Acknowledgments

I would like to express my sincere gratitude to the members of the SSRC Executive Committee for selecting me as the recipient of the 2022 Lynn. S. Beedle Award. I am sure there are other equally if not more deserving individuals. As such, I am very appreciative of this outstanding honor.

I have been fortunate to be a part of numerous research, development, and education activities tied to the discussions in this paper. Some of the earliest activities, initiated by and enhanced within the Structural Stability Research Council, are described in the paper introduction. The following list highlights a few subsequent activities, crediting at least some of the essential participants and collaborators:

- The SSRC Task Group 29 initiatives on second-order inelastic analysis for frame design, culminating in a 1993 workshop and the SSRC (1993) publication *Plastic Hinge Based Methods for Analysis and Design of Steel Frames,* with a foreword by John Springfield, papers and presentations authored by David Anderson, Leo Argiris, P.K. Basu, Russell Bridge, Will Chen, Murray Clarke, Greg Deierlein, Antonello DeLuca, Ciro Faella, Jerry Hajjar, Greg Hancock, Steve Kennedy, Richard Liew, Bill McGuire, Elena Mele, Nick Trahair, Don White, and Ron Ziemian, and workgroup summary reports by Ted Galambos, John Gross, Eric Lui. and Michael Swanger. This workshop brought more than 50 SSRC colleagues together to discuss the various considerations. The developments were a forerunner of subsequent efforts leading to the AISC 360 Direct Analysis and Advanced Analysis provisions for design.

- The ASCE LRFD Task Committee effort leading to the (ASCE 1997) publication *Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design*, with contributions by Russell Bridge, Murray Clarke, Roberto Leon, Eric Lui, Taqir Sheikh, Don White, and Jerry Hajjar (Chair). This 442-page book provided an early assessment of alternative methods of frame stability design, serving as a precursor to work on the AISC Direct Analysis Method.

- The Joint AISC-SSRC Committee on Frame Stability developments and recommendations, summarized in the (Deierlein et al. 2002) paper "Proposed New Requirements for Frame Stability using Second-Order Analysis," authored by Greg Deierlein, Jerry Hajjar, Joe Yura, Don White, and Bill Baker. Joe Yura and Greg Deierlein Co-Chaired the AISC-SSRC Committee. The committee members were Bill Baker, Ted Galambos, Jerry Hajjar, Rich Henige, Leroy Lutz, Keith Mueller, Shankar Nair, Clint Rex, Robert Tremblay, Don White, and Ron Ziemian. Corresponding members were Reidar Bjorhovde, Duane Ellifritt, Nestor Iwankiw, and Roberto Leon. In addition, significant contributions from outside the committee were provided by Russell Bridge, Will Chen, Murray Clarke, Bill LeMessurier, Bill McGuire, John Springfield, and Andrea Surovek.

- The AASHTO, NCHRP, and FHWA efforts involved with the FHWA Curved Steel Bridge Research Project and the unification of the AASTHO LRFD *Specifications* for straight and curved steel girder bridge design, accomplished from 2002-2005. This effort was led by Ed Wasserman (Chair AASTHO T-14 Committee) and John Kulicki (PI of an NCHRP effort on this topic), with contributions by numerous individuals including Fasil Beshah, Aaron Chang, Charlie Culver, Karl Frank, Mike Grubb, Joey Hartmann, Dan Hall, Se-Kwon Jung, Roberto Leon, Narin Phoawanich, Richard Sause, Bill Wright, Don White, Jay Yoo, John Yadlosky, and Abdul Zureick. Mike Grubb, Bill Wright, and I received the George S. Richardson Medal from the Engineers Society of Western Pennsylvania in 2006 for our efforts in the drafting of the unified AASHTO LRFD provisions for straight and curved I-girder bridges, and assisting the AASHTO T-14 Committee with bringing this significant change forward to the main governing body for the AASHTO LRFD Bridge Design *Specifications*.

- The AISC Task Committee 10 (TC-10) development of the AISC *Specification* Direct Analysis Method (Joe Yura, Chair 1999-2005; Shankar Nair Chair, 2005-2010; Ron Ziemian Chair, 2010-2016), with contributions by numerous individuals. Shankar Nair received the 2007 T.R. Higgins award for his contributions and leadership in bringing the Direct Analysis Method (the DM) to fruition within the *Specification*. Much credit is also due to Joe Yura and Greg Deierlein for their early leadership as co-chairs of the AISC-SSRC Committee on Frame Stability, to Joe Yura for his early leadership and guidance as Chair of AISC TC-10, and to Ron Ziemian for his efforts in shepherding the continued development of the DM as subsequent Chair of TC-10 (and currently of AISC TC-3).

- The AISC Task Committee 4 development of the AISC *Specification* Section F2 to F5 unified provisions for design of I-section members in AISC 360-05 (Lou Geschwindner, Chair 1999-2005), with contributions by numerous individuals. Many of the concepts and considerations behind these advancements are summarized in (White 2008 and 2009).

- The AISC TC-10 Ad hoc Task Group efforts to address the application of a wide range of current and emerging inelastic analysis methods within the AISC *Specification*, leading to Appendix 1 of the ANSI/AISC 360-10 *Specification*. This group was composed of Bill Baker, Greg Deierlein, Subash Goel, Rich Henige, Chris Hewitt, Dick Kaehler, Bill McGuire, Chia-Ming Uang, Don White, and Ron Ziemian (Chair). Appendix 1 has been refined further within the AISC 360-16 and the upcoming AISC 360-22 *Specifications* under Ron Ziemian's leadership as Chair of AISC TC-10 and now AISC TC-3.

- The development of the AISC/MBMA Design Guide 25 (DG 25) 1st Edition (Kaehler et al. 2011). The first edition of Design Guide 25 was authored as a supplement to the 2005/2010 AISC *Specification* to address the design of steel frames using nonprismatic members. Dick Kaehler led the writing of the Guide, with input from the findings gained via Yoon Duk Kim's research (Kim 2010). The second edition of DG 25 is dedicated to Dick, remembering his quiet and steady friendship and leadership in the development of the first edition of this Guide, as well as in numerous technical committee activities for AISC and AISI before his passing in March 2015. It is also dedicated to Yoon Duk, who contributed greatly to the first and second editions of DG 25, and whose passing came all too soon in May 2018.

- The development of AISC Design Guide 28 (DG 28), *Stability Design of Steel Buildings* (Griffis and White 2013). Larry Griffis led the writing of DG 28, providing a wealth of design experience in explaining and illustrating the application of the AISC Effective Length, Direct Analysis and First-Order Analysis methods.

- The improvement of AASHTO LRFD provisions for longitudinally stiffened girder webs, researched by Lakshmi Subramanian (Subramanian and White, 2017) and implemented in the AASHTO LRFD 9th Edition *Specifications* (AASHTO 2020). Lakshmi's research also raised several questions, the solutions of which are addressed in Section 8 of this paper.

- The AASHTO/FHWA initiative on *Proposed LRFD Specifications for Noncomposite Steel Box-Section Members* (White et al. 2019), with substantial contributions from Ajinkya Lokhande, Tony Ream, Charles King, Mike Grubb, Frank Russo as a technical advisor, and the FHWA Program Manager, Brian Kozy, and the subsequent development of significant updates to the AASHTO LRFD 9th Edition *Specifications* (AASHTO 2020) led by the AASHTO T-14 committee, Tom Macioce (Chair). These developments have extended the accuracy and applicability of the AASHTO LRFD rules for various more general steel bridge structures and components, including trusses, arch ribs with unstiffened and longitudinally stiffened webs, tie girders, straddle bents, edge girders in cable-stayed bridges, and steel tower legs. White (2021) provides an extensive overview of the background to these new rules and the other AASHTO (2020) provisions for the design of steel members and structural systems.

- The development of the Design Guide 25 (DG 25) 2nd Edition (White et al. 2021), with significant contributions from Woo Yong Jeong and Ryan Slein as co-authors, and from Oğuzhan Toğay, in updating and improving the SABRE2 software employed in many of the Guide's calculations. The substantial guidance provided by MBMA steering committee members is also gratefully acknowledged.

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References

AASHTO (2020). *AASHTO LRFD Bridge Design Specifications*, 9th Edition, American Association of State and Highway Transportation Officials, Washington, D.C.

AASHTO (2005). *AASHTO LRFD Bridge Design Specifications*, 4th Edition, American Association of State and Highway Transportation Officials, Washington, D.C.

AISC (2022). *Specification for Structural Steel Buildings*, ANSI/AISC 360-22, American Institute of Steel Construction, Chicago, IL (to appear).

AISC (2016). *Specification for Structural Steel Buildings*, ANSI/AISC 360-16, American Institute of Steel Construction, Chicago, IL.

AISC (2010). *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, American Institute of Steel Construction, Chicago, IL.

AISC (2005). *Specification for Structural Steel Buildings*, ANSI/AISC 360-05, American Institute of Steel Construction, Chicago, IL.

ASCE (1997). *Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design*, Technical Committee on Load and Resistance Factor Design, Task Committee on Effective Length, ASCE, 442 pp.

Deierlein, G.G., Hajjar, J.F., Yura, J.A., White, D.W. and Baker, W.F. (2002). "Proposed New Requirements for Frame Stability Using Second-Order Analysis," *Proceedings, Annual Technical Session*, Structural Stability Research Council, 1-20.

Deshpande, A.M., Kamath, A.M., Slein, R., Sherman, R.J., and White, D.W. (2021). "Built-Up I-Section Member Flexural Resistance: Inelastic C_b Effects from Web Shear Post-Buckling and Early Tension Yielding," Research Report to Metal Building Manufacturers Association, American Institute of Steel Construction, and American Iron and Steel Institute, Report No. 21-02, Structural Engineering, Mechanics and Materials, School of Civil and Environmental Engineering, Georgia Institute of Technology, 113 pp.

Dowswell, B. (2018). *Curved Member Design*, AISC Design Guide 33, American Institute of Steel Construction, Chicago, IL, 156 pp.

Gardner, L (2008). "The Continuous Strength Method," *Structures & Buildings*, Proceedings of the Institution of Civil Engineers, SB3, 127-133.

Griffis, L.G. and White, D.W. (2013). *Stability Design of Steel Buildings*, AISC Design Guide 28, American Institute of Steel Construction, Chicago, IL, 175 pp.

Kaehler, R., White, D.W. and Kim, Y.D. (2011). *Design of Frames Using Web-Tapered Members*, AISC/MBMA Design Guide 25, American Institute of Steel Construction, Chicago, IL, 204 pp.

Kim, Y.D. (2010). "Behavior and Design of Metal Building Frames Using General Prismatic and Web-Tapered Steel I-Section Members," Doctoral Dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA., 528 pp.

Latif, W. and White, D.W. (2022). "Flange Local Buckling Resistance and Local-Global Buckling Interaction in Slender-Flange Welded I-Section Beams," *Engineering Journal*, AISC (to appear).

Liang, C., Reichenbach, M.C., Helwig, T.A., Engelhardt, M.D. and Yura, J.A. (2021). "Effects of Shear on the Elastic Lateral Torsional Buckling of Doubly Symmetric I-Beams," *Journal of Structural Engineering*, ASCE, published online on December 29, 2021, DOI: 10.1061/(ASCE)ST.1943-541X.0003294.

Mozer, J., Cook, J. and Culver, C. (1971). "Stability of Curved Plate Girders – P2," Prepared for the Department of Transportation, Federal Highway Administration, and Participating States under Contract Number FH-11-7389, Department of Civil Engineering, Carnegie-Mellon University, Pittsburgh, PA, 111 pp.

Nair, S., Patel, V., Abou, N. and Wilkinson, S. (2019). "Artistic Arch," *Modern Steel Construction*, January.

Nair, S. (2016). "Current Views from Past Higgins Award Winners," North American Steel Construction Conference, Education Archives, AISC, cloud.aisc.org/nascc/2016/N48.mp4.

Nair, S. (1986). "Buckling and Vibration of Arches and Tied Arches," *Journal of Structural Engineering*, ASCE, 112(6), 1429-1440.

Schilling, C.G. (1996). "Yield-Interaction Relationships for Curved I-Girders," *Journal of Bridge Engineering*, ASCE, 1(1), 26-33.
Slein, R. Kamath, A.M., Latif, W., Phillips, M., Sherman, R.J., Scott, D.W., and White, D.W. (2021). "Enhanced Characterization of the Flexural Resistance of Built-Up I-Section Members," Research Report to Metal Building Manufacturers Association, American Institute of Steel Construction, and American Iron and Steel Institute, Report No. 21-01, Structural Engineering, Mechanics and Materials, School of Civil and Environmental Engineering, Georgia Institute of Technology, 210 pp.
Subramanian, L.P., Jeong, W.Y., Yellepeddi, R. and White, D.W. (2018). "Assessment of I-Section Member LTB Resistances Considering Experimental Tests and Practical Inelastic Buckling Design Calculations," *Engineering Journal,* AISC, 55(1), 15-44.
Subramanian, L.P. and White, D.W. (2017). "Flexural Resistance of Longitudinally Stiffened Plate Girders," Updated Report to American Association of State Highway and Transportation Officials and American Iron and Steel Institute, School of Civil and Environmental Engineering, Atlanta, GA, 392 pp.
SSRC (1993). *Plastic Hinge Based Methods for Advanced Analysis and Design of Steel Frames: An Assessment of the State-of-the-Art*, D.W. White and W.F. Chen (ed.), Structural Stability Research Council, 299 pp.
Trahair, N.S. and Papangelis, J.P. (1987). "Flexural-Torsional Buckling of Monosymmetric Arches," *Journal of Structural Engineering*, ASCE 113(10), 2271-2288.
Watwood, V.B. (1985). "Gable Frame Design Considerations," *Journal of Structural Engineering*, ASCE 111(7), 1543-1558.
White, D.W. (2021). "Strength Behavior and Design of Steel Systems and Members," *Steel Bridge Design Handbook*, National Steel Bridge Alliance, American Institute of Steel Construction, Chicago, IL, 289 pp.
White, D.W., Jeong, W.Y. and Slein, R. (2021). *Frame Design using Nonprismatic Members*, AISC/MBMA Design Guide 25, 2nd Ed., American Institute of Steel Construction, Chicago, IL, 405 pp.
White, D.W., Slein, R. and Toğay, O. (2020). "Advancements in the Stability Design of Steel Frames Considering General Nonprismatic Members," *Steel Construction – Design and Research*, February, https://doi.org/10.1002/stco.201900048.
White, D.W., Lokhande, A., Ream, A., King, C. and Grubb, M.A. (2019). "Proposed LRFD Specifications for Noncomposite Steel Box-Section Members," Report No. FHWA-HIF-19-063, Office of Bridges and Structures, Federal Highway Administration, Washington, DC., September, 373 pp.
White, D.W., Jeong, W.Y. and Toğay, O. (2016). "Comprehensive Stability Design of Planar Steel Members and Framing Systems via Inelastic Buckling Analysis," *International Journal of Steel Structures*, 16(4), 1029-1042.
White, D.W. (2009). "Unified Design of Steel I-Section Flexural Members," T.R. Higgins Lecture, North American Steel Construction Conference, Education Archives, AISC, cloud.aisc.org/nascc/2009/videos/NASCC2009_K2.mp4.
White, D.W. (2008). "Unified Flexural Resistance Equations for Stability Design of Steel I-Section Members – Overview," *Journal of Structural Engineering*, ASCE, 134(9), 1405-1424.
White, D.W. and Grubb, M.A. (2005). "Unified Resistance Equations for Design of Curved and Tangent Steel Bridge I-Girders," *Proceedings*, *TRB 6th International Bridge Engineering Conference*, Transportation Research Board, Boston, MA, July, 121-128.
White, D.W., Zureick, A.H., Phoawanich, N.P. and Jung, S.K., (2001). "Development of Unified Equations for Design of Curved and Straight Steel Bridge I Girders," Final Report to AISI, PSI Inc. and FHWA, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, October, 547 pp.
Yoo, C.H. and Davidson, J.S. (1997). "Yield Interaction Equations for Nominal Bending Strength of Curved I-Girders," *Journal of Bridge Engineering*, ASCE, 2(2), 37-44.