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Finite element model-based dynamic characteristic predictions for cold-formed steel storage racks

Umeshkumar T. Mourya[1], S. Arul Jayachandran[2]

Abstract
Cold-formed perforated uprights form the backbone of the storage racks most widely used for storage and warehouse systems. Prediction of the fundamental time period of a rack structure is of importance for its design subjected to dynamic loading. A simplified procedure to estimate the time period is available in codes of practice, and the same has been evaluated with the results of a rigourous finite element model. The improved model takes into account the semi-rigidity of the connections at the base and the beam-to-upright connections, which are generally ignored by practicing engineers for the simplicity of the design process. The results indicate a need to propose improved amplification factors to increase the suitability of simplified procedures in time period estimations, thus avoiding the need for preliminary analysis for time-period calculations.

1. Introduction
Selective pallet racks are the most commonly preferred type of structure in warehouses. These structures are assembled from cold-formed steel sections, which are very slender in nature. The column member is generally a mono-symmetric stiffened channel section, generally referred to as upright, forms the basic component of the racking system. These uprights are braced together to form an upright frame. The upright frames are used in the cross-aisle direction, providing stability by braced frame action and connected by pallet beams in the down-aisle direction. Figure 1 shows a general arrangement of a storage rack with all the components. Plan bracings also connect the two down-aisle frames at various levels to provide stability against torsion. Due to the absence of bracing in down-aisle direction, the front frame has to rely on the moment frame action to stabilize the structure. Sometimes, spine bracing is provided in the back frame to provide additional stability, but the frame governs the behavior of the structure without the bracings.

Moreover, these racks are preferred owing to the convenience of quick assembly and de-mounting of the overall system. This is achieved through pallet beams provided with hook-in end connectors. A typical beam-to-upright connection is shown in Fig. 2. The drawback of the hook-in end connectors is that they are not rigidly connected to the upright, allowing independent beam rotation to certain extent inducing semi-rigidity in the structural behaviour of these racks. Taller frames are becoming increasingly popular these days, to make better use of the available storage

[1] Research Scholar, Indian Institute of Technology Madras, <umourya.iitm@gmail.com>
[2] Professor, Indian Institute of Technology Madras, <aruls@iitm.ac.in>

space. Hence it is important to analyze and design these structures with greater precision and semi-rigidity.

The dynamic behaviour of structures is governed by the structure's fundamental time period of vibration. The analysis/response of storage racks is very complicated due to perforations in the uprights, semi-rigidity of the beam-to-upright connections, and the semi-rigid connection at the base.

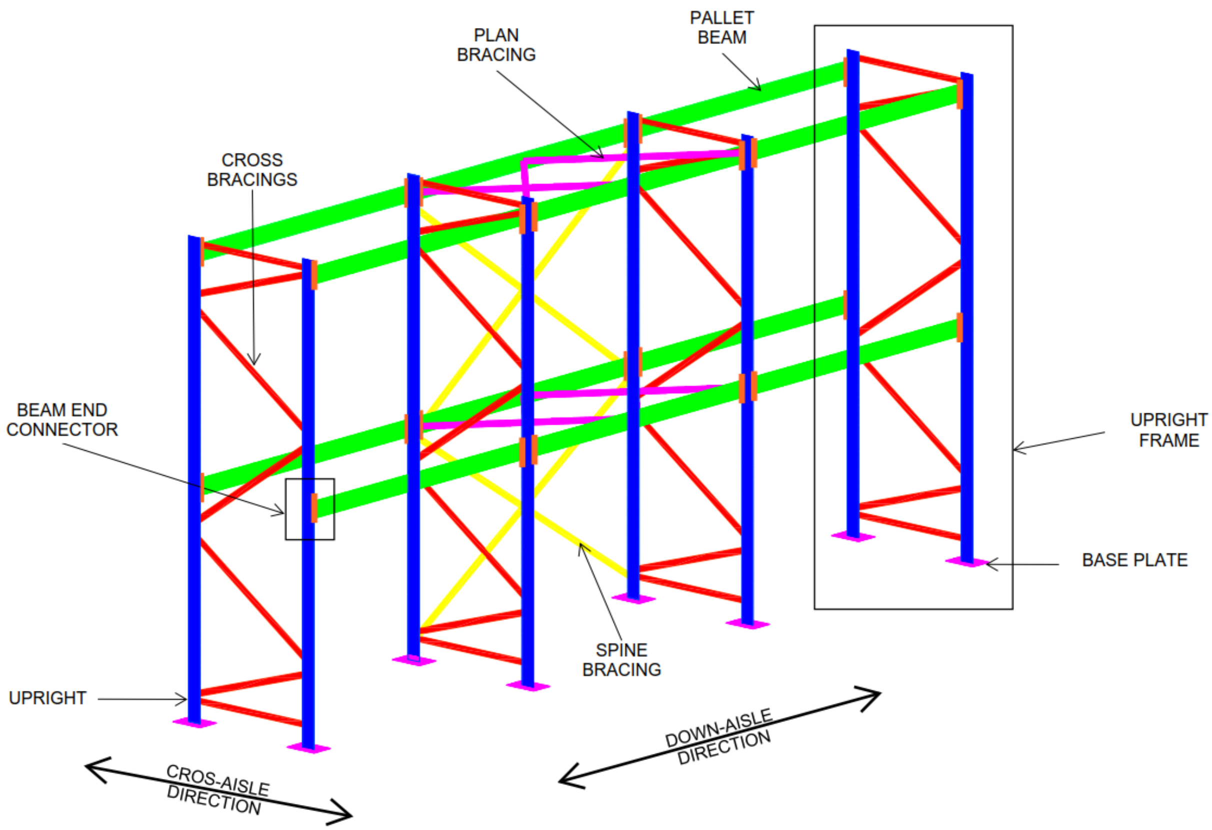

Figure 1: General arrangement of a storage rack

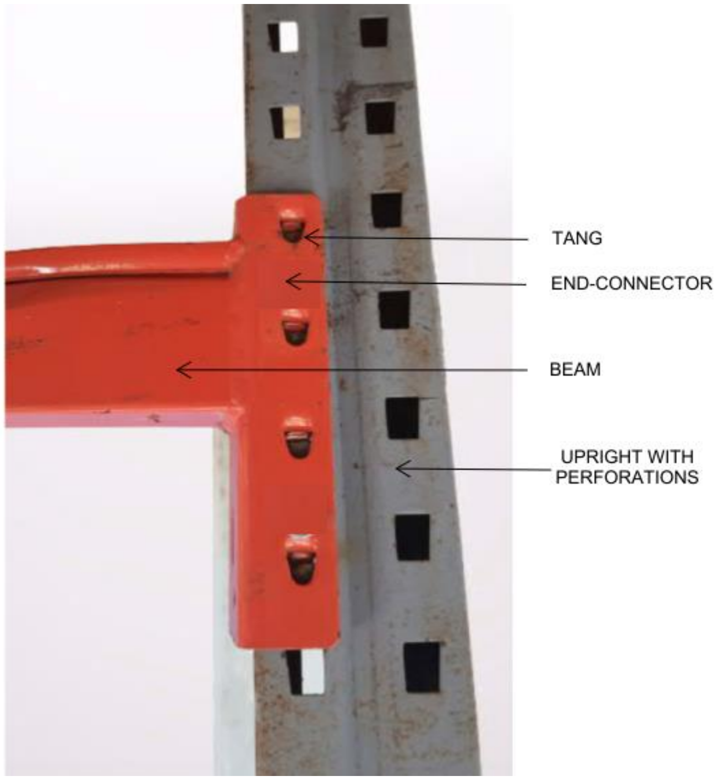

Fig. 2: Typical boltless beam to upright connection.

2. Review of literature

2.1 Beam-to-upright connections

A review of the literature has been conducted on the behavior of beam-to-upright connections. Markazi et al. (1997) conducted tests on commercially available beam-to-upright end connectors and classified them into different classes. Prabha et al. (2010) experimentally studied various configurations and identified the important parameters like the thickness of the upright, depth of the beam, and the depth of the end connector, for characterization of these end connectors. Zhao et al. (2014) and Mohan et al. (2015) studied the failure modes of different end connectors. Other researchers such as Shah et al. (2015), Escanio et al. (2018), and Dumbrava and Cerbu (2020) have also conducted tests on various configurations and obtained the moment-rotation relationship of end connectors. The initial stiffness of the connections from these articles has been summarized in Figure 3. More than 120 data points were collected, and the statistical analysis yields the following observations:

1. The minimum and maximum values of connection stiffness show a wide variation between 19.8 kN-m/rad and 240 kN-m/rad.
2. 20% of the cases fall below 50 kN-m/rad, 51% cases lie between 50 –100 kN-m/rad, 18% cases lie between 100 – 150 kN-m/rad, and only 11% cases lie between 150 – 200 kN-m/rad. Overall 69% cases lie between 50 – 100 kN-m/rad.

3. From the data points plotted in Figure 3, for the data points lying between 50 – 100 kN-m/rad, the average value of the connection stiffness is found to be 85.6 kN-m/rad, and the average value of the connection stiffness of all the data points is 87.7 kN-m/rad.

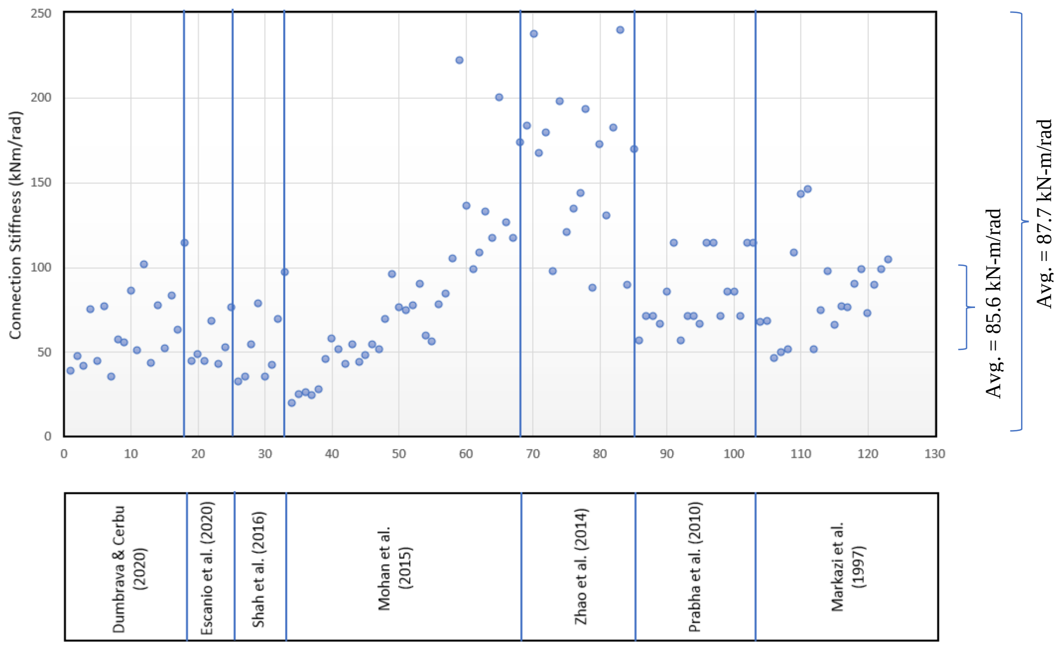

Figure 3: Beam-to-upright connections literature

2.2 Time period estimation

Based on Rayleigh's method, RMI 2012 recommends the approximate formula shown in Eq. 1 (T_1^R). As an alternative to this method, FEMA 460 recommends a formula based on the displacement-based model available in literature Filiatrault et al. (2006), shown in Eq. 2. This method accounts for the semi-rigid connections at the base and the beam-to-upright connections and does not require a preliminary structural analysis to estimate the period. The authors noted that the proposed formula gives good results for beam-to-upright connection stiffnesses ranging from 100-1000 kN-m/rad.

$$T_1^R = 2\pi\sqrt{\frac{\sum W_i \delta_i^2}{g\sum F_i \delta_i}} \qquad (1)$$

$$T_1^F = 2\pi\sqrt{\frac{\sum_{i=1}^{N_{LL}} W_i\, h_i^2}{g\left[N_{btc}\left(\frac{K_{btc}\cdot K_b}{K_{btc}+K_b}\right) + N_{base}\left(\frac{K_{base}\cdot K_u}{K_{base}+K_u}\right)\right]}} \qquad (2)$$

Bernuzzi et al. 2015 presented a study on time period estimations and proposed the following suggestion:

i) To use second-order analysis to estimate the time period for Rayleigh's method (T_1^{RA}), or use amplification factors proposed therein. The amplification factor (α_{cr}) is shown in Eq. 4.

ii) The authors also investigated the efficacy of FEMA method and proposed to consider second-order effects through amplification factor, shown in Eq. 6.

$$T_1^{RA} = \beta^{RA} \cdot T_1^R \tag{3}$$

$$\beta^R = \left(\frac{1}{1-\alpha_{cr}}\right)^{\psi} \tag{4}$$

$$T_1^{FA} = \beta^F \cdot T_1^F = \sqrt{1+\alpha^F} \cdot T_1^F \tag{5}$$

$$\alpha_{cr}^F = \frac{\sum_{i=1}^{N_{LL}} W_i\, h_i \cdot \left(\frac{K_{btc}+K_b}{K_{btc}K_b}\right)}{\left[N_{btc} + N_{base}\left(\frac{K_{base}K_u}{K_{btc}K_b}\right)\left(\frac{K_{btc}+K_b}{K_{base}+K_u}\right)\right]} \tag{6}$$

Arul Jayachandran (2019) performed experimental and numerical studies on dynamic behaviour of storage racks. The natural frequencies of the storage rack with and without masses were obtained from sweep sine tests.

2.3 Limitations of existing methods

It was found in Bernuzzi et al. 2015 that the beam-to upright connections stiffnesses range from 200 kN-m/rad to 4000 kN-m/rad (the latter is close to fixed end condition). The connection stiffnesses, as experimentally observed, generally lie between 20 – 200 kN-m/rad, as can be observed in Fig. 3. This difference suggests a need for investigation of the applicability of existing equations for time period estimates. Consequently, this study investigates the existing equations to the applicability of connections stiffness values between 20 – 200 kN-m/rad which encountered in most real world situations.

3. Numerical Analysis

Portal frames of storage racks in the down-aisle direction have been modeled. The parameters considered are the total height of the structure and the stiffness of the beam-to-upright connections. Frames were chosen to have 5 and 10 levels, each with three bays, for this study. The bay width of frames is 2.5m, and the level height is 1.5m. Figure 4 shows the typical frame model used in this study. Base connection stiffness of 70 kN-m/rad has been considered in the analysis. The initial stiffness of connections of 20, 50, 100, 150 and 200 kN-m/rad have been considered in this study. Material properties of F_y = 355 N/mm^2 and E = 2 × 10^5 N/mm^2 were considered in the analysis. Table 1 shows the relevant properties of the sections used in the study. The results and conclusions have been presented in the subsequent sections. Various configurations used in the study are shown in Table 2.

EN 15512 (2009) prescribes that the frame imperfection (ϕ) should include out of plumb of the frame and the effect of looseness of the beam-to-column connection. A minimum value of out of plumb ϕ = 1/500 is recommended for the total frame imperfection, and the maximum is limited to 1/350. The imperfection due to looseness have to be obtained from the test conducted as prescribed in EN 15512 (2009). A total imperfection ϕ = 1/300 has been considered. A typical gravity load of 2kN/m has been considered for analysis. This represents general loading pattern in storage racking industry. This loading has been applied to the beam in all the bays at the stories. Notional horizontal loads have been considered at all the beam levels as EN 15512 (2009) recommended to account for the effect of frame imperfection.

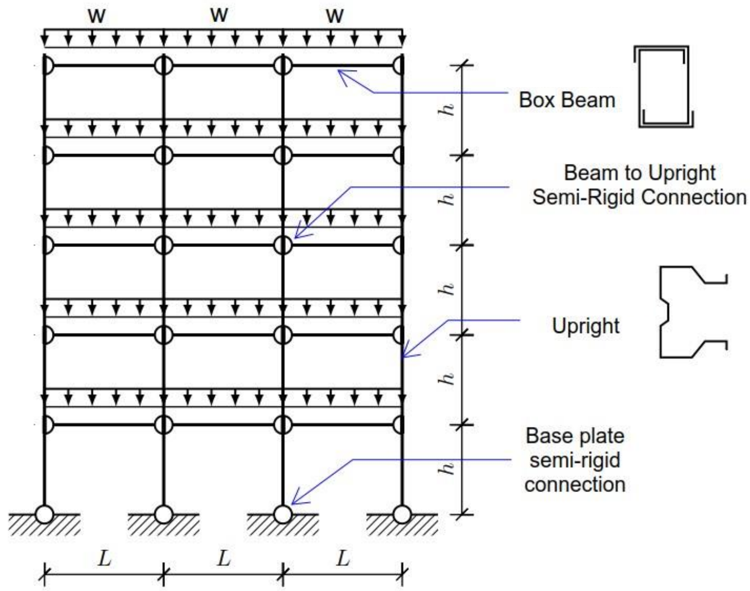

Figure 4: Model used in the study

Table 1: Sectional properties

S. No.	Name	A	I_{zz}	I_{yy}	Designation
1	70x70x6x1.5	369.51	2.56E+05	1.99E+05	T1
2	90x70x6x1.5	412.50	4.52E+05	2.23E+05	T2
3	110x70x6x2.0	572.68	1.02E+06	3.17E+05	T3
5	Box 50x80x1.6	405.76	3.67E+05	1.77E+05	B1
5	Box 50x100x1.6	469.76	6.29E+05	2.15E+05	B2
6	Box 50x140x1.6	597.76	1.45E+06	2.90E+05	B3

4. Methodology

The different frame configurations used in this study are shown in Table 2. The frames have been analyzed in MASTAN2 v5. The following methodology has been adopted:

i) Elastic critical buckling analyses have been performed to obtain the buckling load factor for the frames under the applied loads. The objective was to ensure that the buckling load factor is greater than one, i.e., the frame does not buckle in sway mode under the applied loads.

ii) Modal analysis has been performed in the second step to obtain the fundamental time period (T_{A1}) of the frames under the applied gravity loads.

iii) Second-order analysis has been performed in the third step to obtain the geometric stiffness matrix and the amplified drifts.

iv) Finally, modal analysis has been performed again, but considering the geometric stiffness matrix and the elastic stiffness matrix to obtain the time period of the frames (T_{A2}).

v) The fundamental time period of the frames is obtained from Eq. (2) (T_M), and the amplified time period (T_{MA}), accounting for the second-order effects, is also obtained from Eq. (5) and (6).

vi) The amplification factor required has been calculated as the ratio of time period obtained from second-order analysis stiffness matrix to time period from the simplified method.

vii) The time periods obtaied from the various methods have been compared and presented in Table 2.

5. Results and Discussion

Various frames were modelled to obtain the fundamental time periods considering the first and second-order effects. The results are as follows:

i) The time period of the frames obtained from the simplified model has been found to be lesser than the actual time period obtained modal analysis.

ii) The time periods based on second-order effects, i.e., accounting geometric stiffness matrix are highly sensitive to the level of loading. The amplification in the time period increases as the buckling load factor approaches unity.

iii) There is a significant difference in amplification factor available in the literature and the actual required in storage racks. The amplification factor predicted by existing equations ranges from 6 to 21%, whereas the current study shows that this factor needs to be in the range of 32 to 55%, based on the frames analyzed in this study. The primary factor is the difference between the connection stiffnesses considered in previous studies and the actual range found in practice.

6. Conclusions

The following conclusions have been drawn from the results obtained in this study:

i) The simplified model in FEMA-460 underestimates the time period of the frames as compared to the actual time period obtained from the modal analysis using a stiffness matrix based on first-order analysis.

ii) The amplification factor proposed in the code is also found to be inadequate. This is primarily due to the fact that uprights are generally mono-symmetric sections and are more prone to cross-sectional deformations, which add to the second-order deformations.

iii) Since the uprights are mono-symmetric sections, the time periods based on second-order effects (i.e., accounting geometric stiffness matrix) are highly sensitive to the loading level. The amplification in the time period increases as the buckling load factor approaches unity.

iv) A more comprehensive study is needed to propose improvements in the amplification factor for time period estimation using simplified equations available in literature and code of practices.

v) Experimental studies are required to generate database of time-periods of storage racks. These results can be used to calibrate numerical models and thus propose accurate amplification factors for use with simplified equations.

Table 2: Results from Analysis

S. No.	N bays	N storey	Frame Config.	$k_{connection}$ (kN-m/rad)	T_M (sec)	T_{A1} (sec)	T_{A2} (sec)	T_M / T_{A1}	T_M / T_{A2}	Buckling Load factor	Amplification factor	Amplified Period (T_{MA})	Amplification factor required
1	3	5	T1B1	20.0	3.236	3.487	5.024	0.928	0.64	1.707	1.214	3.928	1.55
2	3	5	T1B1	50.0	2.350	2.659	3.211	0.884	0.73	2.715	1.118	2.627	1.37
3	3	5	T1B1	100.0	1.878	2.251	2.571	0.834	0.73	3.525	1.077	2.022	1.37
4	3	5	T1B1	150.0	1.677	2.086	2.339	0.804	0.72	3.930	1.062	1.781	1.39
5	3	5	T1B1	200.0	1.564	1.995	2.217	0.784	0.71	4.169	1.054	1.648	1.42
6	3	5	T2B2	20.0	3.169	3.302	4.474	0.960	0.71	1.950	1.109	3.513	1.41
7	3	5	T2B2	50.0	2.250	2.464	2.877	0.913	0.78	3.219	1.109	2.495	1.28
8	3	5	T2B2	100.0	1.743	2.023	2.245	0.861	0.78	4.393	1.066	1.858	1.29
9	3	5	T2B2	150.0	1.520	1.835	2.000	0.828	0.76	5.060	1.051	1.597	1.32
10	3	5	T2B2	200.0	1.391	1.728	1.868	0.805	0.75	5.487	1.043	1.451	1.34
11	3	10	T3B2	20.0	6.27	6.635	-	0.945	-	0.882	-	-	-
12	3	10	T3B2	50.0	4.33	4.757	7.062	0.910	0.61	1.567	1.201	5.199	1.63
13	3	10	T3B2	100.0	3.31	3.772	4.688	0.878	0.71	2.309	1.122	3.719	1.41
14	3	10	T3B2	150.0	2.88	3.349	3.953	0.859	0.73	2.795	1.094	3.148	1.37
15	3	10	T3B2	200.0	2.63	3.109	3.579	0.846	0.73	3.138	1.079	2.838	1.36
16	3	10	T3B3	20.0	6.18	6.576	-	0.940	-	0.898	-	-	-
17	3	10	T3B3	50.0	4.17	4.614	6.632	0.904	0.63	1.693	1.188	4.953	1.59
18	3	10	T3B3	100.0	3.09	3.563	4.310	0.867	0.72	2.531	1.107	3.421	1.40
19	3	10	T3B3	150.0	2.61	3.098	3.563	0.843	0.73	3.151	1.078	2.815	1.36
20	3	10	T3B3	200.0	2.33	2.827	3.174	0.825	0.73	3.609	1.062	2.476	1.36

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