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Brace Stiffness Quantification for Lean-on Bracing

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Abstract

Cross-frames are critical for the stability of steel bridges during construction and play an important role in completed bridges. Historically, brace locations have been regions of fatigue concerns, and each brace requires significant handling and processing during fabrication. The braces represent one of the most expensive bridge components per unit weight. Therefore, there are major benefits to minimizing the number of cross-frames in a bridge in terms of economics and structural performance. In the application of lean-on bracing concepts, select cross-frames are replaced in certain bracing lines with top and bottom struts, which allow a single cross-frame to brace several girders as a method of minimizing the number of cross-frames in a bridge. Lean-on concepts were developed for the Texas Department of Transportation (TxDOT) in the early 2000s. Previous studies developed design guidelines, but recent applications of lean-on bracing in TxDOT bridge designs demonstrated the need for improved efficiency and clarity. The primary focus of this paper is related to the stability stiffness and strength capacity of lean-on cross-frame lines. The impact of the number of cross-frames per bracing line, will be discussed in terms of stability implications. Future work will validate the approaches detailed in this paper to determine an expression for the stiffness and strength contribution of the torsional brace.

1. Introduction

I-shaped girders are often utilized in steel bridge systems as an efficient and economical solution in a wide range of bridge applications. Steel, as a material, has exceptional strength-to-weight properties, and steel girders provide significant flexibility in terms of shipping since the bridge girders can be fabricated in shorter lengths, shipped to the site, spliced together, and quickly erected. However, the high strength-to-weight ratio can lead to slender elements and systems,

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which may prove to be troublesome during erection and other construction phases when the bracing conditions are highly variable. During construction stages, the steel section alone generally supports the full load. Construction stages are generally critical for lateral-torsional buckling (LTB), which is a limit state that involves lateral movement of the compression flange and twist of the section as depicted in Figure 1. Stability in the finished bridge is rarely a concern since the cured concrete deck provides continuous lateral and torsional bracing to the composite system.

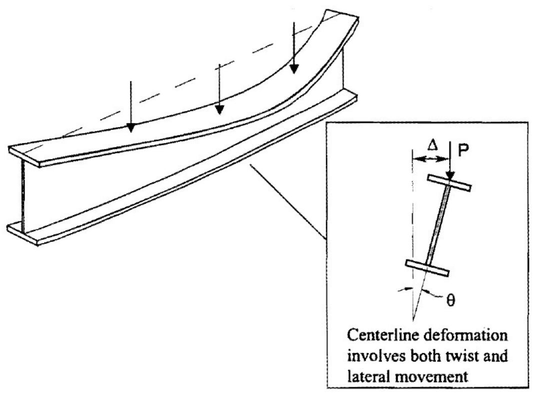

Figure 1. Lateral-Torsional Buckling. Adapted from Helwig and Wang (2003).

An increase in LTB capacity is achieved by providing adequate bracing. Effective beam bracing can be achieved by either restraining lateral displacement of the critical compression flange (lateral bracing), or by controlling twist of the section (torsional bracing). Once the composite concrete deck has cured, the deck and shear studs provide continuous lateral and torsional restraint to the girder top flange and additional stability to the bottom flange. As a result, conventional LTB is not typically a concern in the completed bridge. As alluded to above, the critical stages for LTB are typically during erection and deck placement. In bridge I-girder systems, cross-frames and diaphragms commonly serve as stability braces during construction to enhance the LTB resistance of the girders. Since cross-frames and diaphragms restrain twist of the cross-section at discrete locations along the length of the bridge girder, they are categorized as point (discrete) torsional braces. Though the braces are necessary for girder stability and other functions such as restraining fascia girders from torsion applied by deck overhang brackets, they introduce some complexities into the design and require strategic placement along the length and width of the framing system. These complexities range from difficulties during fabrication and erection to concerns regarding fatigue performance of the girder system. Due to the significant handling and fabrication requirements, the braces are often the most expensive component of steel bridges per unit weight. Therefore, it is advantageous to refine the design and detailing of cross-frame systems.

The AASHTO LRFD Bridge Design Specification (BDS) (2020) provides design, detailing, and analysis guidance for cross-frames and diaphragms, but this guidance is primarily limited to the fatigue limit state. Current AASHTO LRFD has no formal guidance on stability bracing requirements of cross-frames and diaphragms, though a recent study that investigated the stability bracing characteristics of conventional cross-frames in steel I-girder systems resulted in several ballots that were approved in June of 2021 (Reichenbach et al. 2021). Therefore, the next edition of the AASHTO LRFD BDS will include the stability bracing requirements. However, due to the absence of formal design requirements in all current and previous editions of the AASHTO BDS, the typical practice has been to utilize standard brace details and layouts that are specified by state departments of transportation. The braces in straight bridges have historically often not been sized for any specific force or demand. For example, a common member size in cross-frames specified by many bridge owners is an L4x4x3/8 angle.

For cross-frames in steel bridges, conventional detailing practice is to provide braces between adjacent girders across the full width of the bridge as shown in Fig. 2 (A). However, in some applications, such a layout can lead to large live-load induced forces as well as difficulty installing bracing, particularly in bridges with significant support skew. Instead of providing cross-frames across the full width of the bridge, selectively positioning cross-frames within the bridge cross-section and using top and bottom struts to lean other girders on the braced locations, as depicted in Fig 2 (B), can provide improved behavior and efficiency. This concept is referred to as lean-on bracing and has long been applied for bracing of frames for a variety of structural engineering applications. In the early 2000s, lean-on concepts were adapted for implementation into steel I-girder bridges (Helwig and Wang 2003). Lean-on braces offer a cost-effective solution by combining the versatility of a torsional bracing system with the simplicity of a lateral brace. In these systems, torsional braces (typically in the form of cross-frames) are strategically placed throughout the bridge and provide the primary source of stability to the girders.

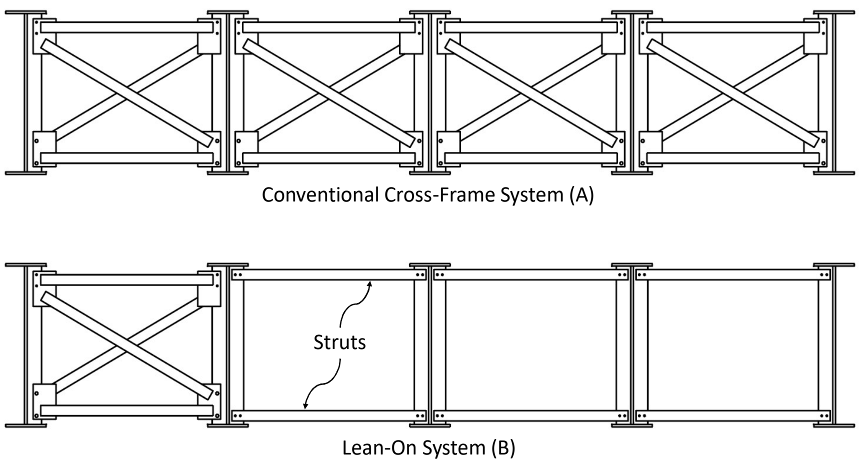

Figure 2. Conventional Cross-Frame System (A) versus a Lean-On System (B)

As noted in the previous discussion, the current AASHTO LRFD provides no guidance on the design of cross-frames for stability bracing requirements. The recently approved AASHTO ballots focus on the stability bracing requirements for conventional bracing; however, lean-on bracing concepts will not be provided in the 10^{th} edition of the AASHTO LRFD BDS. Methods have been developed and documented that aid designers, but some of the simplifications included in the original research may be overly conservative in some instances and unconservative in others. Furthermore, there has been an abundance of research conducted over the past few decades with respect to LTB and the bracing characteristics of cross-frames, but studies related to lean-on systems have been limited outside of the original investigation and implementation (Helwig and Wang 2003; Romage 2008). Therefore, it is important to the design industry to address these potential shortcomings and refine the design procedures for wider applications of lean-on bracing.

Additionally, questions about best practices for incorporating the effects of girder continuity, non-prismatic girder sections, and multiple cross-frames in a given bracing line have recently arisen.

Though these do not comprise a complete list of the areas that may benefit from refinement, they do serve as an initial scope for the background studies reviewed in this research. The focus of this research is to study the behavior of lean-on bracing systems with a goal of refining and improving guidance that will allow designers to more easily implement lean-on bracing into steel bridge designs. If properly detailed and distributed along the bridge length and width, lean-on bracing systems can reduce the number of full cross-frames required while potentially improving the long-term bridge behavior. A well-designed and detailed lean-on bracing system potentially offers significant savings in fabrication costs, simplifies the erection process, and can alleviate in-service cross-frame force demands in heavily skewed bridges. In short, strategic use of lean-on braces can serve as an efficient alternative to traditional cross-frame systems.

In previous work, equations were developed for the brace stiffness of a typical bracing system and for a lean-on system with only one cross-frame in the brace line. As a result, this paper outlines an ongoing study of three approaches for the stiffness and force distribution in lean-on systems with multiple cross-frames per line.

2. Background

In order to understand the proposed approaches for lean-on bracing with multiple cross-frames per line, it is necessary to begin with the required brace stiffness of the torsional bracing system. The current equations for the provided brace stiffness of a typical bracing system are then discussed.

2.1 Brace Stiffness Requirement Equation

The recent ballot provisions for inclusion into the AASHTO bridge design specifications are generally an extension of the required brace stiffness provided in AISC (2017), and given in the following expression:

$$\beta_{T\,req} = \frac{2.4LM_u^2}{\varphi nEI_{eff}C_b^2} \tag{1}$$

where L is the span length, M_u is the factored design moment, φ is 0.75 (LRFD), n is the number of intermediate braces, E is the modulus of elasticity, C_b is the moment gradient factor, and I_{eff} is defined as:

$$I_{eff} = I_{yc} + \frac{t}{c}I_{yt} \tag{2}$$

where I_{yc} is the lateral moment of inertia of the compression flange, I_{yt} is the lateral moment of inertia of the tension flange, t is the distance from the centroid of the tension flange to the neutral bending axis, and c is the distance from the centroid of the compression flange to the neutral bending axis.

The expression shown in Eq. 1 approximately provides twice the ideal stiffness and is assumed to limit the twist at the brace location to a value equal to the initial imperfection, θ_0. Therefore, the resulting brace moment (M_{br}) is given by the following expression:

$$M_{br} = \beta_{T\,req'd}\theta_0 = \frac{2.4LM_u^2}{\varphi nEI_{eff}C_b^2}\frac{L_b}{500h_0} \tag{3}$$

2.2 Cross-Frame Stiffness Equation

The provided brace stiffness must meet or exceed the required brace stiffness:

$$\beta_T \geq \beta_{Treq'd} \tag{4}$$

where β_T is the total brace stiffness of the torsional system and is generally a function of three stiffness components. Most stability bracing systems follow the equations for springs in series as given by the following expression:

$$\frac{1}{\beta_T} = \frac{1}{\beta_b} + \frac{1}{\beta_g} + \frac{1}{\beta_{sec}} \tag{5}$$

where β_b is the stiffness of the brace, β_g is the in-plane girder stiffness, and β_{sec} is the stiffness of the cross section related to cross sectional distortion. Eq. 5 indicates that β_T is less than the smallest of the three individual stiffness components, which are assumed to interact as springs in series. From this relationship, it is evident that an otherwise stiff cross-frame can be adversely affected by poor in-plane girder stiffness or significant distortional effects in the girder webs. Thus, the overall stiffness of a torsional brace is effectively limited by the most flexible component in Eq. 5.

2.3 In-Plane Girder Stiffness, β_g

The in-plane (i.e., vertical) flexural stiffness of the bridge girders themselves contribute to the overall stiffness of the torsional bracing system. The stiffness contribution of the girders was first shown in twin-girder systems (Helwig et al. 1993). As shown in Fig. Figure 3, when the girders are subjected to a twist, the internal moment in the cross-frame is equilibrated by vertical shear forces acting at the ends of the brace. The vertical forces on the adjacent girders cause one girder to deflect upwards and the other to deflect downwards leading to a rigid body rotation. These deformations reduce the effectiveness of the brace. With a wider system, this displacement is reduced, as demonstrated by the four-girder system shown in the same figure.

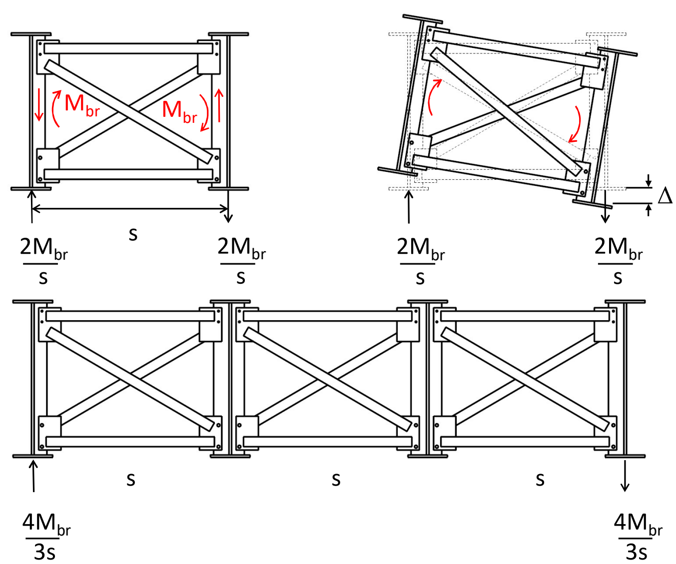

Figure 3. In-Plane Girder Stiffness.

This behavior was quantified for a twin-girder system in Eq. 6 (Helwig et al. 1993):

$$\beta_g = \frac{12s^2EI_x}{L^3} \tag{6}$$

Where E is the modulus of elasticity, I_x is the in-plane moment of inertia of the girder, and L is the span length. For a framing system with more than two girders, Eq. 7 is instead used (Yura 2001; Helwig and Yura 2015):

$$\beta_g = \frac{24(n_g-1)^2s^2EI_x}{n_gL^3} \tag{7}$$

where n_g is the number of girders in the system. The in-plane girder stiffness contribution is most critical in narrow systems, such as two or three-girder bridges, and is tied to a mode of buckling that is often referred to as the system buckling mode (Yura et al. 2008; Han and Helwig 2016). If β_g is less than $\beta_{T\,req'd}$, full bracing cannot be achieved regardless of the stiffness of the stiffness of the brace that is utilized. From a buckling perspective, the system mode will control over buckling between the brace points. As noted previously, design guidance for the system failure mode has been incorporated into AASHTO LRFD (2020). A modified expression for the in-plane stiffness has been developed and proposed by Fish (2021), which is based upon the system buckling model.

Helwig and Wang (2003) recommended the reduction of the in-plane girder stiffness by 50% when utilizing lean-on bracing as compared a system only utilizing traditional torsional bracing concepts, as expressed in Eq. 8. It is assumed that a lean-on system would have an in-plane girder stiffness between that of a twin-girder system and of a traditional cross-frame layout, and finite element analysis solutions showed reasonable correlation with the 50% reduction. However, this equation can potentially be adjusted for increased precision and although not covered in this paper, is being investigated in the present research study.

$$\beta_g = \frac{12(n_g-1)^2s^2EI_x}{n_gL^3} \tag{8}$$

2.4 Cross-Section Stiffness, β_{sec}

If the braces are relatively shallow compared to girder depth, the stiffness of the cross-section, β_{sec}, may have a significant effect. Yura and Helwig (2015) derived Eq. 9 for full-depth web stiffeners when the distance from the top cross-frame to the top of the girder is the same as than the distance from the bottom of the cross-frame to the bottom of the girder. This form is included in AISC Appendix 6:

$$\beta_{sec} = \frac{3.3E}{h_w}\left(\frac{(1.5h_w)t_w^3}{12} + \frac{t_sb_s^3}{12}\right) \tag{9}$$

where h_w is the height of the web, t_w is the thickness of the web, t_s is the thickness of the stiffener, b_s is the width of the stiffener. The first term in the equation accounts for the effective moment of inertia for the part of the web assumed to participate in the distortion, and the second term accounts for the moment of inertia of the stiffener, taken about the centroid of the web.

Only the region outside of the brace depth contributes to the cross-sectional distortion. Because most cross-frames in bridge I-girder applications are relatively deep with respect to the girder depth, the cross-section stiffness component tends to be a large value, such that it is not usually a significant concern in Eq. 5. Language in the approved ballot for AASHTO allows β_{sec} to be taken as infinity for braces deeper than 80% of the web depth, which is relatively common in most bridges. This provision recognizes the significant stiffness for relatively deep braces. As a result, β_{sec} can often be ignored.

2.5 Torsional Brace Stiffness, β_b

Torsional bracing systems typically utilize cross-frames or diaphragms to help bridge girders resist LTB. Cross-frames can be found in the form of X-shapes, K-shapes, and occasionally Z-shapes. (Fig. 4) X-type braces work well with deep girders, such as in built-up I-girder bridges, while K-type braces or diaphragms are better suited for shallower girders. The torsional stiffness (i.e., the stiffness response of the brace when subjected to an in-plane moment) of the brace can be estimated based on an idealized truss model (Yura 2001; Helwig and Wang 2003). Equations have been derived for each brace type and the derivation process is discussed in the next section.

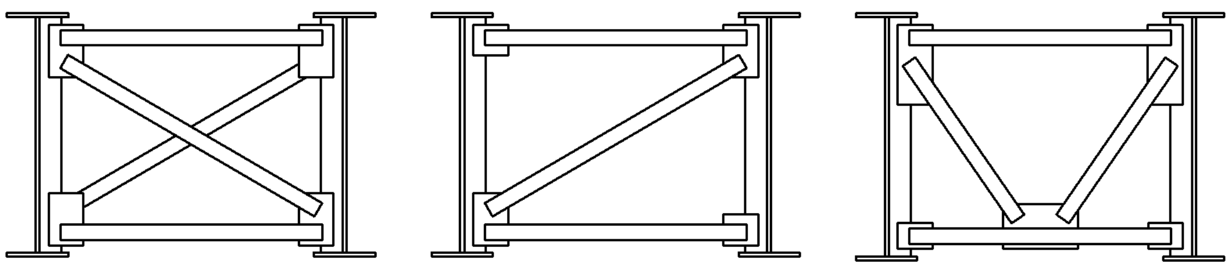

Figure 4. Various Forms of Cross-Frames.

3. Current Torsional Brace Stiffness Derivations

Two particularly relevant derivations are accepted for bracing stiffness: one for a single cross-frame, and one for a cross-frame line with a single cross-frame and lean-on struts. Both are discussed in the following sections.

3.1 Twin Girder System Derivation

Yura (2001) developed an equation to estimate the torsional brace stiffness of a Z-type cross-frame or a tension-only X-type cross-frame, for which the compression diagonal is conservatively neglected (assuming that member might buckle). In many cases, cross-frames are constructed with slender angle sections whose compression load-carrying capacity is relatively small and therefore neglected. Virtual work was used to derive the expression. The idealization of this system is shown in Fig. 5.

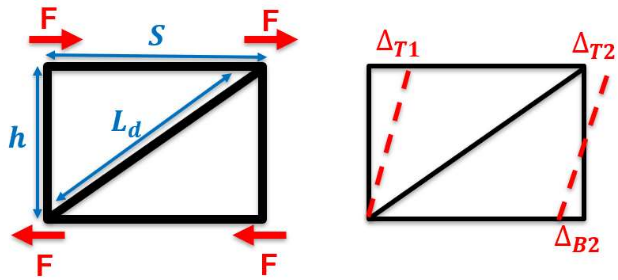

Figure 5. Twin Girder Brace Stiffness Idealization

In this approach, Eqns. 10 and 11 are combined to result in Eqn. 12, indicating that the displacement of the critical girder is the basis for calculating the provided stiffness.

$$M = Fh \tag{10}$$
$$\beta_b = \frac{M}{\theta} \tag{11}$$
$$\beta_b = \frac{Fh^2}{\Delta_{crit}} \tag{12}$$

where M is the moment applied to the system, F is a unit load applied at the top and bottom of each girder in the directions shown, h is the height of the brace, θ is the rotation of the girder, and Δ_{crit} is the critical displacement of the girder (here, $\Delta_{T2} + \Delta_{B2}$). From the virtual work procedure, Δ_{crit} is calculated, resulting in Eqn. 13 for β_b. This is the equation currently accepted for typical bracing.

$$\beta_b = \frac{h^2 S^2 E}{\frac{2L_d^3}{A_d} + \frac{S^3}{A_S}} \tag{13}$$

where S is the girder spacing, h_b is the depth of the cross-frame, L_d is the length of the cross-frame diagonal members, $A_d$is the cross-sectional area of the cross-frame diagonals, and A_h is the cross-sectional area of the cross-frame struts.

From Eq. 13, it is evident that the stiffness of the brace is a function of the axial stiffness of its individual members when the cross-frame is subjected to a moment. Although not explicitly presented, the inherent flexibility of the connections should also be considered in the evaluation of the overall brace stiffness, similar to what is done for cross-section distortional effects or in-plane girder flexibility.

For many cross-frame applications, single-angle or tee sections are attached to connection or gusset plates along only one leg or flange, respectively. This, in turn, introduces an eccentric load path through the connection that can significantly impact the stiffness of the brace. In lieu of a more refined assessment of these softening effects, AASHTO LRFD (2020) recommends a simple reduction factor based on experimental and analytical studies conducted by Battistini et al. (2013; 2016) and Wang (2013). For stability bracing applications, a fixed factor of 0.65 can be applied to the cross-sectional area of the diagonals and struts in the development of Eq. 13. This reduction factor was calibrated to represent these softening effects for a wide range of common cross-frame configurations, connections, and member sizes.

3.2 Lean-On Bracing Derivation

Helwig and Wang (2003) derived a generalized equation for the brace stiffness contribution in a lean-on bracing system based on the idealization developed by Yura (2001). The expression is:

$$\beta_b = \frac{h^2 S^2 E}{\frac{n_{gc} L_d^3}{A_d} + \frac{S^3}{A_S}(n_{gc} - 1)^2} \tag{14}$$

where n_{gc} is the number of girders per cross-frame. In this expression, the number of cross-frames per bracing line is assumed to be one, so n_{gc} is effectively the number of girders. As an example, the idealization of a four-girder system is shown in Fig. 6. The freebody diagram shows the accumulation of forces that develop across the width of the bridge. The bracing demand from the girders results in force couples that lead to the forces indicated in the figure. Some designs that have made use of Eq. 14 have included more than one brace in a given line, which results in an erroneous estimate of the stiffness demand since the resulting value of n_{gc} in those cases is not representative of the force distribution across the cross-frame line.

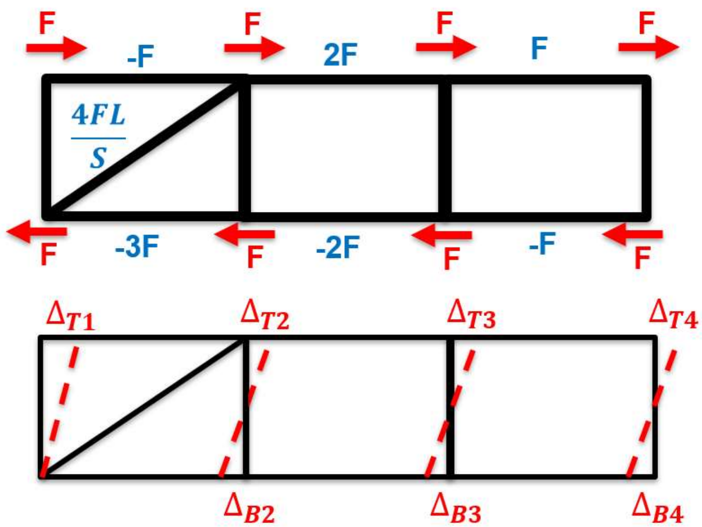

Figure 6. Lean-On Bracing Stiffness Idealization

The calculated brace stiffness of this system is shown in Eq. 15

$$\beta_b = \frac{h^2 S^2 E}{\frac{4 L_d^3}{A_d} + \frac{9 S^3}{A_S}} \tag{15}$$

4. Approaches for Lean-On Systems with More than One Cross-Frame per Bracing Line

Currently, designers that have utilized lean-on bracing concepts often make use of more than one cross-frame in each line in their application of lean-on bracing. Using more than one brace per line is done in an attempt to reduce the demand on the cross-frame. However, the resulting n_{gc} that is used is not representative of the stiffness derivation for Eq. 14. It is therefore necessary to develop an expression for the brace stiffness that accounts for the additional cross-frame relative to the

Helwig and Wang expression in Eq. 14. Three potential approaches are described in the following sections in the context of a four-girder system. In the future, the best approach will be generalized to account for various bridge configurations.

4.1 Cross-Section Slice Approach

The first approach is the "Cross-Section Slice," where a redundant cross-frame is essentially ignored. This is shown for a four-girder system with two cross-frames in Fig. 7, where the left cross-frame is not considered in determining the brace stiffness. This results in Δ_{crit} equal to the sum of $\Delta_{T3}, \Delta_{T4}, \Delta_{B3}, and\ \Delta_{B4}$. The brace stiffness of the configuration is then calculated using Eq. 15 or virtual work for a three-girder system, resulting in Eq. 16.

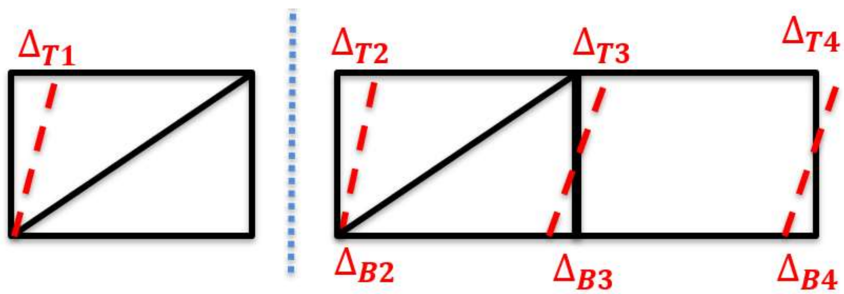

Figure 7. Cross-Section Slice

$$\beta_b = \frac{Fh^2}{\Delta_{crit}} = \frac{Fh^2}{\frac{3FL_d^3}{S^2 A_d E} + \frac{4FS}{A_s E}} \tag{16}$$

4.2 Displacement Combination Approach

The "Displacement Combination" approach is similar to the "Cross-Section Slice," but the displacement of the second girder is accounted for differently. In the "Cross-Section Slice," the displacement is ignored, whereas in "Displacement Combination" the displacement is accounted for by adding the second girder from the first cross-frame to the result obtained in the "Cross-Section Slice." This is shown in Fig. 8, and the resulting stiffness is Eq. 17. In this approach, Δ_{crit} is equal to the sum of $\Delta_{T2}, \Delta_{T3}, \Delta_{T4}, \Delta_{B2}, \Delta_{B3}, and\ \Delta_{B4}$.

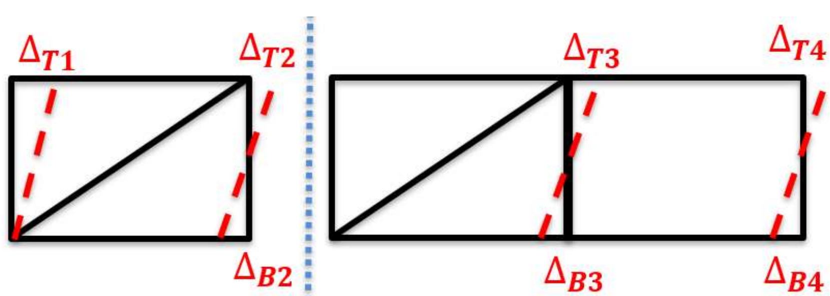

Figure 8. Displacement Combination

$$\beta_b = \frac{Fh^2}{\Delta_{crit}} = \frac{Fh^2}{\frac{5FL_d^3}{S^2 A_d E} + \frac{5FS}{A_s E}} \tag{17}$$

4.3 Stiffness Superposition Approach

The "Stiffness Superposition" is the least conservative of the three approaches. In this idealization, the cross-frame line is essentially broken into two single-cross-frame systems, and the respective stiffnesses of each system are added together. This is shown for the four-girder, two-cross-frame system in Fig. 9, with the resulting brace stiffness equation shown in Eq. 18.

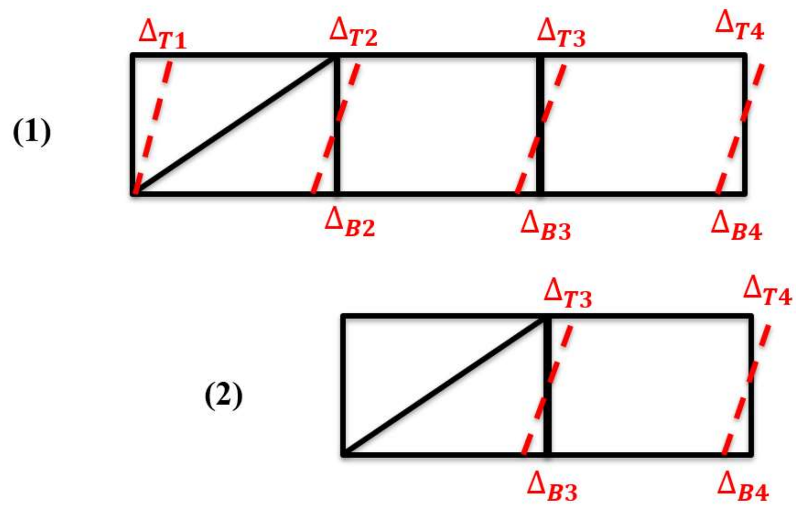

Figure 9. Stiffness Superposition

$$\beta_b = \beta_{b1} + \beta_{b2} = \frac{Fh^2}{\frac{4FL_d^3}{S^2 A_d E} + \frac{9FS}{A_S E}} + \frac{Fh^2}{\frac{3FL_d^3}{S^2 A_d E} + \frac{4FS}{A_S E}} \quad (18)$$

5. Comparison of Results

The impact of each approach was quantified for realistic bridge geometries based on an existing bridge utilizing lean-on bracing. The bracing stiffness was calculated for a bridge with four girders spaced at 2.4 meters (8 feet) and 3.0 meters (10 feet), with two cross-frames tested with depths of 1.75 meters (69 inches) and 2.4 meters (96 inches). The impact of each of the three approaches was similar for all four configurations. The results for 2.4 meter (8 foot) spacing and 1.75 meter (69 inch) cross-frame depth are shown in Table 1 for the brace stiffness, as well as the overall bracing system stiffness based on constant in-plane girder and cross-section stiffnesses. The results are normalized against the Helwig and Wang equation (Eq. 15) for only one cross-frame.

Table 1: Comparison of Results

Approach	β_b	β_t
Helwig and Wang – 1 CF	1.00	1.00
Cross-Section Splice	1.72	1.06
Displacement Combination	1.67	1.05
Stiffness Superposition	2.72	1.10

The values in the table indicate that the various approaches can result in an increase of up to 10% in the total provided stiffness. The Stiffness Superposition approach results in the most significant increase in the brace stiffness, since the stiffness of two systems are added together, resulting in some redundant bays. The Cross-Section Splice and Displacement Combination approaches result in similar values, with the Displacement Combination approach being the more conservative. This makes sense because a greater critical displacement is considered in that approach due to how the second girder is accounted for. The three different methods are being considered with respect to accuracy as well as simplicity in application

6. Conclusion and Ongoing Work

In this work, a gap was identified in the existing literature for lean-on bracing design with multiple cross-frames per bracing line. Equations have been accepted for typical bracing and lean-on bracing with one cross-frame. There is no existing expression for the brace stiffness provided by lean-on systems with multiple cross-frames in a bracing line.

In response to this knowledge gap, three approaches to quantify the brace stiffness of these systems were developed: Cross-Section Slice, Displacement Combination, and Stiffness Superposition. Initial comparisons were made by applying these approaches to a typical bridge section. The Displacement Combination method was the most conservative, while the Stiffness Superposition method was the least.

In future work, all three approaches will be evaluated using finite element analysis in order to determine an appropriate equation for this design scenario. Model displacements will be used to calculate the stiffness of the system, and the best approximation will be selected. The approximation will then be generalized to apply to different numbers of cross-frames and braces.

Acknowledgements

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