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Quantifying error associated with simplifications to the direct analysis method

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Abstract
All methods of design for stability strike a balance between analytical accuracy and simplicity. The direct analysis method defined in the AISC *Specification* is a robust method of design that has been calibrated and validated to a range of numerical and experimental results. Use of the direct analysis method is advantageous for engineers because it eliminates the need to calculate effective length factors. However, aspects of the method, such as the application of notional loads and stiffness reductions, can be cumbersome in practice. Simplifications to reduce design burden exist but could be expanded with evidence showing that the expanded simplifications do not compromise safety. Thus, data showing how potential simplifications affect the analytical accuracy of methods of stability design is needed. The objective of this study is to generate the necessary data for several potentially beneficial simplifications. Differences in maximum permitted applied loads are evaluated between methods of design with and without the simplifications. Differences are quantified for a suite of simple frames, each featuring a single structural steel beam-column but with different member slenderness, boundary conditions, and leaning column load. The results indicate that some simplifications such as neglecting stiffness reductions are not safe in general but can be employed safely for defined ranges of structures and conditions. These simplifications, once implemented after further development into design provisions, will enable engineers to focus their efforts on other aspects of the design.

1. Introduction

The direct analysis method defined in Chapter C of the American Institute of Steel Construction (AISC) *Specification for Structural Steel Buildings* (AISC 2022) has rules for calculation of required strengths and for calculation of available strengths. Required strengths must be calculated using a second-order analysis, initial system imperfections must be considered, and adjustments to stiffness must be made. While the use of second-order analysis is generally not problematic because many modern structural analysis software packages have this capability, the other two requirements can be cumbersome or inconvenient to apply in design.

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Methods of design for stability have evolved over time, but it has long been recognized that second-order effects, geometric imperfections, and partial yielding affect the strength of structural steel frames. Thus, geometrically and materially nonlinear analysis with imperfections included, commonly abbreviated as GMNIA, often serves as the best approximation of true behavior and is used to produce benchmark data to assess methods of design based on simpler analysis approaches. Important studies that developed and validated the effective length method (Kanchanalai 1977) and the direct analysis method (Surovek-Maleck and White 2004) used GMNIA in this way.

Putting safety first is a fundamental principle of engineering. Accordingly, the development of new methods of design tends to focus on safety and limiting unconservative error. Accounting for geometric imperfections in an analysis to determine required strengths will never be unsafe. Yet, second-order effects, geometric imperfections, and partial yielding don't affect all structural steel frames equally. Because of this, further development of methods of design often focuses on identifying simplifications that enable engineers to perform design more efficiently while maintaining safety.

The AISC *Specification* (2022) includes several exceptions and simplifications to methods of design for stability. Section C2.2b(d) allows initial system imperfections to be neglected in load combinations with lateral load if the ratio of maximum second-order drift to maximum first-order drift is below a certain limit. Section 7.2.3(b) allows the use of an effective length factor of one for moment frames if the ratio of maximum second-order drift to maximum first-order drift is below a different limit.

Engineers recognize when their effort to implement certain design provisions has little to no impact on the result. Often, specific provisions of the direct analysis method have no effect on the final design of moment frames since the design of moment frames is often governed by drift limit requirements. During the 2021 Innovations for Research-to-Industry Stability Engagement (I-RISE) Summit hosted by the Structural Stability Research Council and AISC, engineers communicated their frustration with some provisions of the direct analysis method. Thus, even though some exceptions and simplifications exist, more would be helpful and more can likely be developed.

Furthermore, some existing exceptions and simplifications may not be conveyed as well as they could be. The main benefit of the direct analysis method is the use of the unbraced length, L, as the effective length, L_c (equivalently expressed as setting the effective length factor, K, equal to 1). Yet, AISC *Specification* (2022) Section 7.2.3 allows the use of $K = 1$ for braced frames and moment frames when the ratio of maximum second-order drift to maximum first-order drift is less than 1.1. Engineers seeking to optimize their design process will use these rules to avoid the stiffness reduction requirements in the direct analysis method. A more consistent and uniform approach may be to have an exception to the stiffness reductions in the direct analysis method under certain circumstances. Nonetheless, any simplification must be rigorously justified with data. Recognizing the need, AISC funded a research project to investigate potential improvements to the direct analysis method. This study is among the first steps of that research project.

In this study, several potential simplifications to the direct analysis method are evaluated to quantify the error that they introduce. Error is measured by differences in maximum permitted applied loads between methods of design with and without the simplifications and for many small frames, each featuring a single structural steel beam-column. In this approach, the methods with simplifications are compared to methods that were calibrated to the results of GMNIA, but not to results of GMNIA directly. While the simplifications investigated in this study would be unsafe if applied to all cases, the data presented can serve as a guide to crafting provisions that enable their use as broadly as possible without compromising safety.

2. Methods

Methods of design are evaluated in this study for the simple generic sidesway uninhibited frame shown in Fig. 1. The frame has a single structural steel beam-column with rotational springs at the top and bottom, a leaning column, and a simply supported beam connecting the columns. The frame and all members are braced against out-of-plane deformations and buckling. Slenderness of the beam-column is controlled by selection of the cross section and length, L. Rotational restraint of the beam-column, which in a moment frame would be provided by the beams framing into the beam-column, is controlled by the stiffness of the rotational springs at the top and bottom of the beam-column, $k_{\theta,top}$ and $k_{\theta,bot}$, respectively. The magnitude of leaning column load is controlled by the parameter γ. This frame has been used in previous studies on methods of design (Denavit 2021; Denavit et al. 2016) and is an abstracted version of frames used in other studies (Kanchanalai 1977; Surovek-Maleck and White 2004).

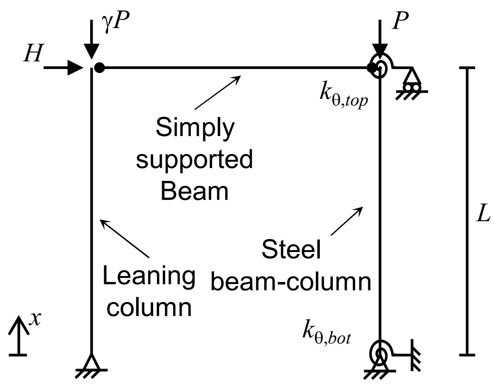


Figure 1: Schematic of frame used in this study

Comparisons between methods of design are made in this study at the applied load level, thus pairs of factored applied loads P and H must be calculated such that they are at a maximum while also satisfying the strength design requirement given by the interaction equation of AISC *Specification* (2022) Section H1.1, presented here as Eqs. 1a and 1b noting that load and resistance factor design (LRFD) is used and the beam-column is subject to single-axis flexure.

When $P/P_n \geq 0.2$

$$\frac{P}{\phi P_n} + \frac{8}{9}\frac{M_2}{\phi M_n} \leq 1.0 \tag{1a}$$

When $P/P_n < 0.2$

$$\frac{P}{2\phi P_n}+\frac{M_2}{\phi M_n}\leq 1.0 \tag{1b}$$

where ϕP_n is the design compressive strength calculated according to AISC *Specification* (2022) Chapter E for the limit state of flexural buckling in the plane of the frame and using $L_c = L$ and a resistance factor $\phi = 0.9$; ϕM_n is the design flexural strength calculated according to AISC *Specification* (2022) Chapter F for the limit state of flexural yielding for bending in the plane of the frame and using a resistance factor $\phi = 0.9$; and M_2 is the required flexural strength.

The required flexural strength, M_2, is the maximum moment in the structural steel beam-column as determined from a second-order elastic analysis performed according to the requirements of the method of design under consideration. Second-order elastic analyses are performed in this study using closed-form solutions to Eq. 2.

$$v''''(x)+\frac{P}{EI^*}v''(x)=0 \tag{2}$$

where v is the lateral deflection of the beam-column and EI^* is the flexural stiffness of the beam-column, including adjustments where required.

Closed-form solutions to Eq. 2 were developed in previous work using a computer algebra system and implemented in MATLAB to enable quick evaluation (Denavit et al. 2016). Quick evaluation is necessary to enable iterative back-calculation of the largest applied loads, P and H, that satisfy Eqs. 1a and 1b. Note that Eq. 2 only accounts for flexural deformations, but that flexural deformations dominate the response of the frames investigated in this study. Closed-form solutions to Eq. 2 were also used to compute the ratio of maximum second-order drift to maximum first-order drift, Δ_2/Δ_1.

2.1 Base Direct Analysis Method

A base version of the direct analysis method is used in this study as a benchmark against which simplifications to the direct analysis method are measured. In this base version of the direct analysis method required strengths are computed using second-order elastic analysis; initial system imperfections are considered by applying a notional lateral load of $0.002P$ (i.e., the lateral load in the analysis is $H + 0.002P$); a factor of 0.8 is applied to the stiffness of the beam-column and the rotational springs; and an additional factor of τ_b given by Eqs. 3a and 3b is applied to the flexural stiffness of the beam-column.

When $P/P_y \leq 0.5$

$$\tau_b = 1.0 \tag{3a}$$

When $P/P_y > 0.5$

$$\tau_b = 4\left(P/P_y\right)\left(1-P/P_y\right) \tag{3b}$$

where P_y is the cross-sectional compressive strength equal to F_yA_g where F_y is the yield stress and A_g is the gross area of the member.

AISC *Specification* (2022) Chapter C currently allows some simplifications to the direct analysis method. For example, the notional load need not be applied in combination with lateral loads when the ratio of maximum second-order drift to maximum first-order drift, Δ_2/Δ_1, is equal to or less than 1.7. Results are presented both with the use of this existing simplification and without it.

2.2 Alternate Direct Analysis Methods

Five alternate versions of the direct analysis method are evaluated in this study. The alternate versions, described in Table 1, include simplifications related to the notional load, adjustments to stiffness, or both.

Table 1: Details of alternate direct analysis methods

Method	Notional Load	Beam-Column Flexural Stiffness	Rotational Spring Stiffness
Base	$0.002P$	$0.8\tau_bEI$	$0.8k_{\theta,top}$ and $0.8k_{\theta,bot}$
Alternate 1	$0.002P$	$0.8EI$	$0.8k_{\theta,top}$ and $0.8k_{\theta,bot}$
Alternate 2	$0.002P$	τ_bEI	$k_{\theta,top}$ and $k_{\theta,bot}$
Alternate 3	$0.002P$	EI	$k_{\theta,top}$ and $k_{\theta,bot}$
Alternate 4	None	$0.8\tau_bEI$	$0.8k_{\theta,top}$ and $0.8k_{\theta,bot}$
Alternate 5	None	EI	$k_{\theta,top}$ and $k_{\theta,bot}$

Alternate methods 1, 2, and 3 are used to investigate the effect of not applying the adjustments to stiffness required by the AISC *Specification* (2022). Note that according to AISC *Specification* (2022) Section C2.3(c), τ_b can be neglected (i.e., set equal to unity) if additional notional load is applied. Alternate method 1 simply neglects τ_b and does not apply additional notional load. Alternate method 2 neglects the 0.8 reduction factor but still includes the τ_b reduction factor. The 0.8 reduction factor applies to both the beam-column flexural stiffness and the stiffness of the rotational springs since they both contribute to the stability of the structure. Alternate method 3 neglects both the 0.8 and τ_b reduction factors; thus, the nominal stiffness of the beam-column and rotational springs are used in the second-order elastic analysis. Alternate method 4 is used to investigate the effect of not considering initial system imperfections (i.e., no notional load). Alternate method 5 is used to investigate the effect of not applying the adjustments to stiffness and not considering initial system imperfections. These alternate versions of the direct analysis method are not safe for all cases and should not be used generally in practice. However, they can be used safely in some cases and the purpose of this study is to quantify the error associated with the simplifications so that suitable provisions can be developed.

3. Example Frame

Results for an example frame are presented in Fig. 2. The frame has a W12×72 steel beam-column oriented to bend about its major axis. The yield strength of the member is $F_y = 50$ ksi. The member is compact for flexure and nonslender for compression, therefore local buckling does not need to be considered. The length of the columns is 17.8 ft based on a slenderness, $L/r = 40$. The top of the beam-column is fixed against rotation (i.e., $k_{\theta,top} = \infty$), the base of the beam-column is pinned (i.e., $k_{\theta,bot} = 0$), and the frame has no leaning column load (i.e., $\gamma = 0$).

Fig. 2 shows results for two cases. For subplots on the left-hand side (i.e., a, c, and e), the base method and alternate methods 1, 2, and 3 have notional loads equal to 0.002*P*. For subplots on the right-hand side (i.e., b, d, and f), the base method and alternate methods 1, 2, and 3 have notional loads equal to 0.002*P* only when the ratio of maximum second-order drift to maximum first-order drift, $\Delta_2/\Delta_1 > 1.7$. No notional loads are applied when $\Delta_2/\Delta_1 \leq 1.7$ as permitted by AISC *Specification* (2022) Section C2.2b(d), noting that lateral load was applied to the frame when $\Delta_2/\Delta_1 \leq 1.7$.

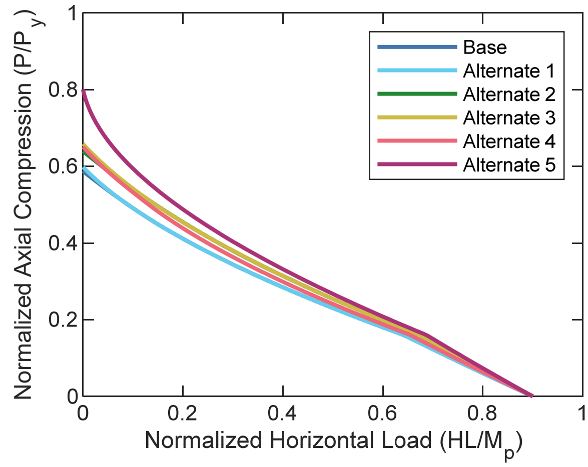
The interaction diagrams shown in subfigures a and b of Fig. 2 represent maximum permitted applied loads for the various design methods. The loads applied to the frame are *P* and *H*. The lateral load *H* is converted to a moment by multiplying by the column length *L* then normalized by the plastic moment, M_p (i.e., the product of the yield stress and the plastic section modulus). The applied load interaction diagram for the base method is the smallest. All the alternate methods permit pairs of applied loads that are not permitted by the base method. This is expected given that the alternate methods have less stiffness reduction, less notional load, or both. When the exception for notional loads for $\Delta_2/\Delta_1 \leq 1.7$ is used, horizontal shifts in the applied load interaction form (Fig. 2b). The horizontal shifts occur when $\Delta_2/\Delta_1 = 1.7$. For lower axial loads (i.e., $\Delta_2/\Delta_1 \leq 1.7$) the notional load is not applied, allowing a larger value of *H* to be applied. The shift occurs at different axial loads for different methods due to the different stiffness reductions applied. For a given frame configuration and flexural stiffness, there is a one-to-one relationship between axial load, *P*, and the ratio of second-order drift to first-order drift, Δ_2/Δ_1. This relationship is shown in Fig. 2c and Fig. 2d. These two figures are identical since Δ_2/Δ_1 does not depend on the notional load. The *x*-axis of the figures was limited to a maximum of 3, but much larger values of Δ_2/Δ_1 were recorded at higher axial loads. Several of the lines overlap since the relationship depends only on the frame configuration and flexural stiffness of the beam-column.

Error results are shown in Fig. 2e and Fig. 2f. The error is computed between the base interaction diagram and the alternate interaction diagrams using a radial measure. For a given angle with respect to the *x*-axis, a radial line is drawn from the origin to the applied load interaction diagrams. The distance from the origin to the interaction diagram for the base is r_{base} and the distance from the origin to the interaction diagram for the alternate design method is $r_{alternate}$ as shown in Fig. 3. The error is computed using Eq. 4.

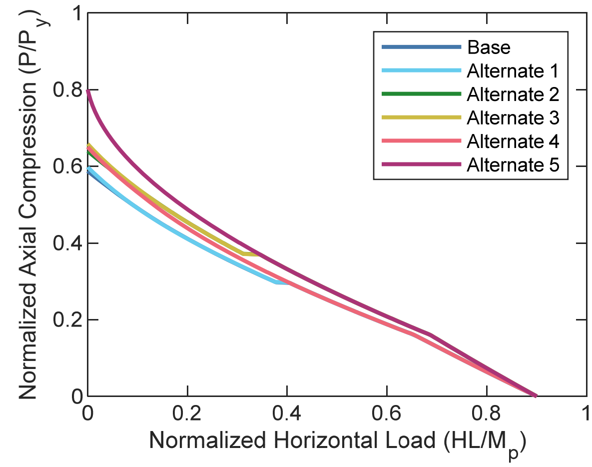
$$\varepsilon = \frac{r_{base} - r_{alternate}}{r_{base}} \tag{4}$$

Negative values of error, ε, indicate that the alternate design method permits applied loads that are deemed unsafe by the base design method, and thus negative error is considered unconservative.

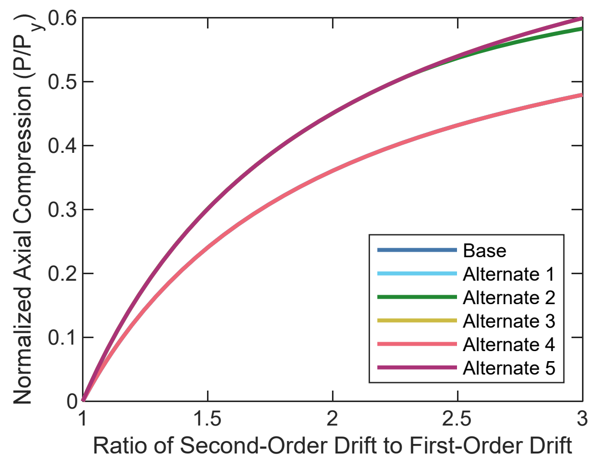
Other definitions of error are possible. For example, the error could be defined as the difference between maximum permitted horizontal loads at a given level of axial load. The value of error tends to be higher using this measure and it may be more reflective of the behavior of moment frames where the axial load in a member is relatively constant. However, using this measure, the error is undefined for axial loads above the maximum permitted by the base method. The radial measure meaningfully quantifies error over the entire interaction diagram.



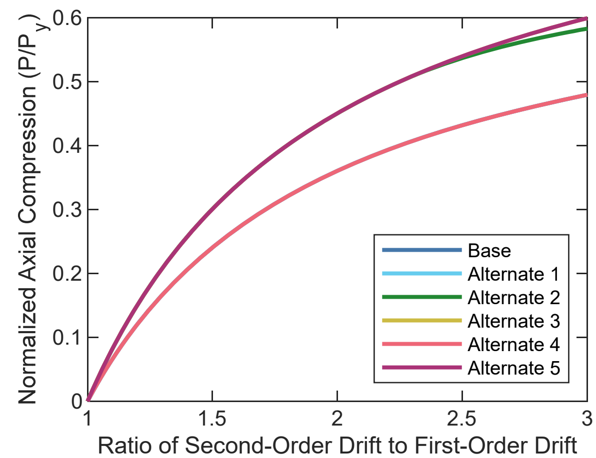
(a) Interaction Diagram



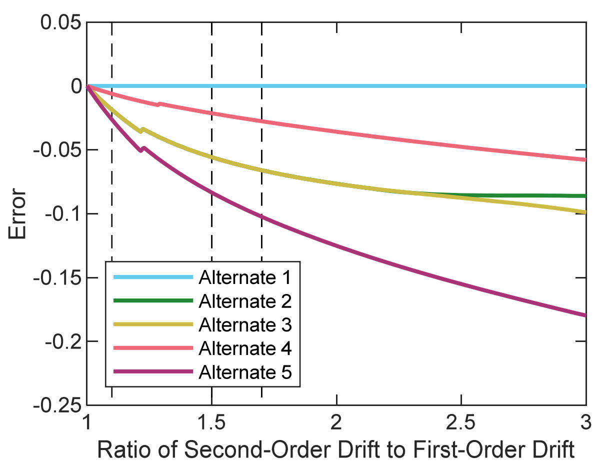
(b) Interaction Diagram
No notional load when $\Delta_2/\Delta_1 \leq 1.7$



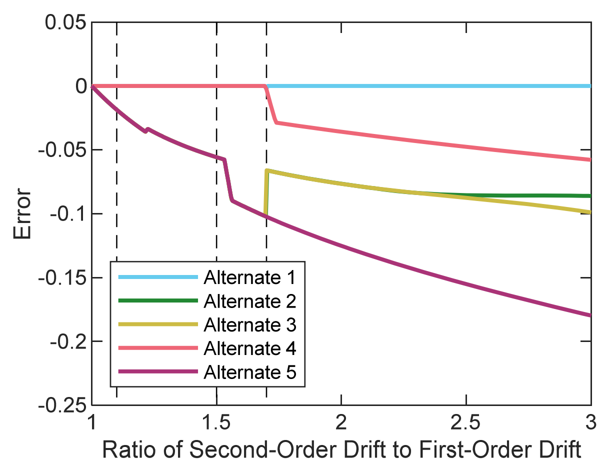
(c) Δ_2/Δ_1 vs P/P_y



(d) Δ_2/Δ_1 vs P/P_y
No notional load when $\Delta_2/\Delta_1 \leq 1.7$



(e) Δ_2/Δ_1 vs Error



(f) Δ_2/Δ_1 vs Error
No notional load when $\Delta_2/\Delta_1 \leq 1.7$

Figure 2: Results for example frame

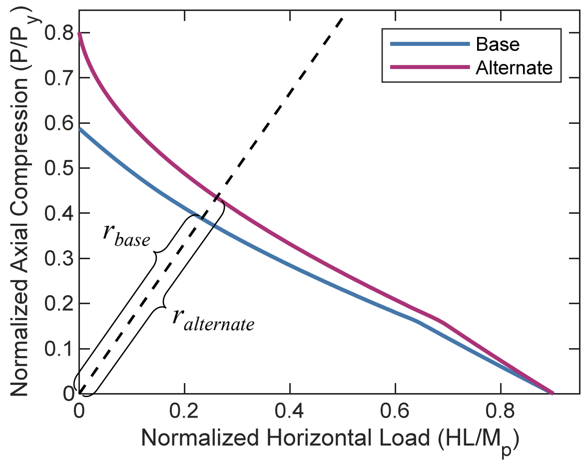


Figure 3: Definition of radial error measure

The error is plotted in Fig. 2e and Fig. 2f as a function of the ratio of second-order drift to first-order drift, Δ_2/Δ_1, computed for the alternate design method at the point on the interaction diagram that corresponds to the angle for which the error was computed. The ratio Δ_2/Δ_1 was computed at each point using the stiffness, including reductions, defined for that point for each method. Vertical black dashed lines at $\Delta_2/\Delta_1 = 1.1$, 1.5, and 1.7 are added to the error plots to indicate key values.

The error for the alternate design methods increases with increasing Δ_2/Δ_1 and thus increasing axial compression. Kinks in the error plots are seen for Δ_2/Δ_1 between about 1.2 and 1.3. These are an artifact of the kink in the bilinear design interaction diagram. When the exception for notional loads when $\Delta_2/\Delta_1 \leq 1.7$ is used, large shifts in the error are seen (Fig. 2f). The errors at lower Δ_2/Δ_1 are also lower for some alternate methods when the exception is used.

4. Parametric Study

The results shown in Fig. 2 provide a detailed view of the effect of simplifications to the base method of design for a single example frame. This section presents results for a range of frames. The same W12×72 beam-column with $F_y = 50$ ksi is used for all cases. However, both major-axis and minor-axis bending are investigated. Slenderness ratios, L/r of 20, 40 and 80 are analyzed, where r is the radius of gyration of the beam-column about its bending axis. Four pairs of rotational spring stiffness values are analyzed as listed in Table 2. Leaning column load ratios, γ of 0, 1, 2, 3 are analyzed. With these variations, 96 different frames are investigated (96 = 2 bending axes × 3 values of L/r × 4 pairs of rotational spring stiffness × 4 values of γ). The range of frames investigated is similar to the range of sway frames investigated by Surovek-Maleck and White (2004) in the original development of the direct analysis method.

Table 2: Selected rotational spring stiffness values

Pair	$k_{\theta,top}$	$k_{\theta,bot}$
1	∞ (fixed)	0 (pinned)
2	$6EI/L$	0 (pinned)
3	∞ (fixed)	∞ (fixed)
4	$6EI/L$	$6EI/L$

The error between each alternate method and the base method is computed as described previously and plotted vs the ratio of second-order drift to first-order drift for the alternate method in Fig. 4 and Fig. 6. Each plot in these figures has 96 lines corresponding to the 96 frames investigated. The lines are colored based on the values of L/r to provide a rough sense of the slenderness of each frame. The results are shown only for $\Delta_2/\Delta_1 \leq 1.5$ to show detail and since the simplifications investigated in this study are less likely to be justifiable for $\Delta_2/\Delta_1 > 1.5$. Subplots on the left-hand side of Fig. 4 and Fig. 6 do not employ the exception for notional load when $\Delta_2/\Delta_1 \leq 1.7$. Subplots on the right-hand side of the figures employ the exception for notional load when $\Delta_2/\Delta_1 \leq 1.7$ defined in AISC *Specification* (2022) Section C2.2b(d). Given the radial error measure used in this study, the error for an alternate method at a particular value of Δ_2/Δ_1 is computed in comparison to the base at a point that may have a different value of Δ_2/Δ_1.

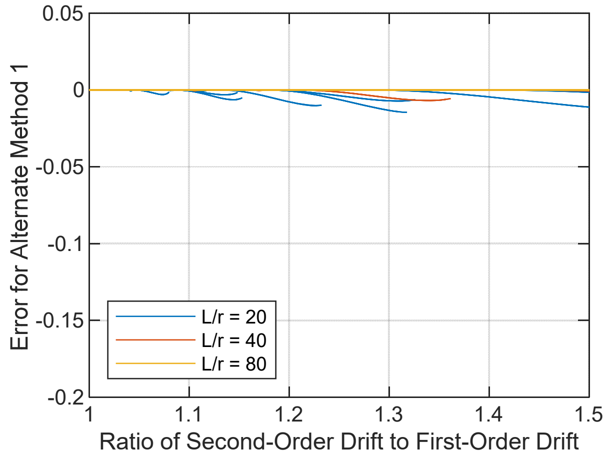
4.1 Effect of Neglecting Stiffness Reductions

The effect of neglecting stiffness reduction is seen by comparing alternate methods 1, 2, and 3 to the base direct analysis method as in Fig. 4. Neglecting the additional stiffness reduction factor τ_b causes minimal error for the frames and range of Δ_2/Δ_1 investigated in this study. Neglecting τ_b only increases the applied load interaction diagram by a few percent and generally for the less slender frames.

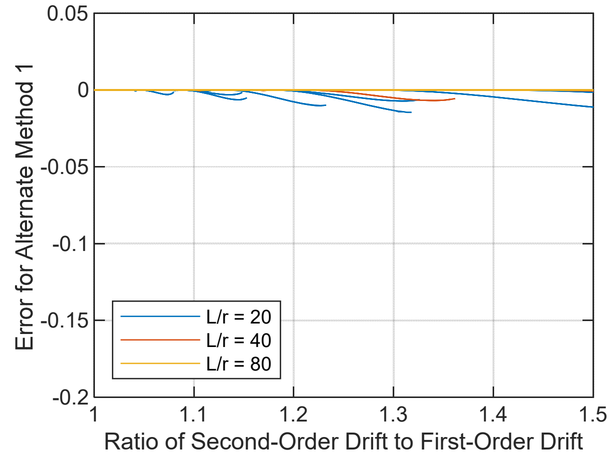
Neglecting the 0.8 stiffness reduction has a greater impact on the maximum permitted applied loads and thus the error is greater. The maximum error increases with increasing Δ_2/Δ_1. At $\Delta_2/\Delta_1 = 1.1$, the maximum error is less than 2.5%. The maximum error reaches 5% at about $\Delta_2/\Delta_1 = 1.25$. 5% is a commonly used limit for unconservative error in methods of design for frame stability (ASCE 1997). The AISC *Specification* (2022) permits design for stability with $K = 1$ and without adjustments to stiffness for moment frames when $\Delta_2/\Delta_1 \leq 1.1$ under the effective length method. The data indicates that the error introduced by this simplification is tolerable.

The maximum error associated with not using the 0.8 stiffness reduction factor for a given Δ_2/Δ_1 occurs for the frames with the slenderest beam-columns. Reducing stiffness increases second-order effects, which are a larger component of the required strength for slender beam-columns. Fig. 5 shows a contour plot of the maximum error for alternate method 3 as a function of Δ_2/Δ_1 and L/r. The maximum errors when $L/r = 40$ are essentially equal to those when $L/r = 80$ up to approximately $\Delta_2/\Delta_1 = 1.3$.

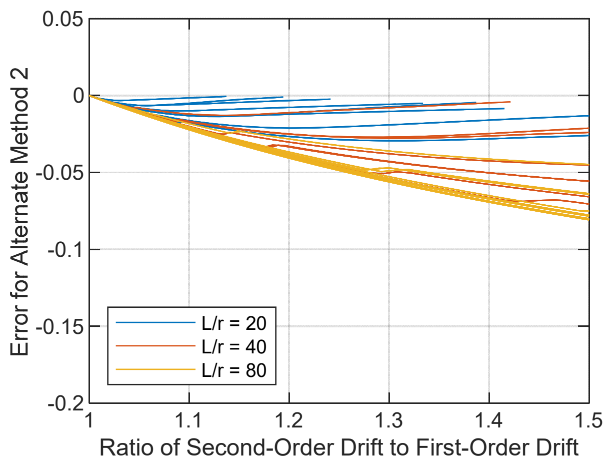
Little difference in error is seen between alternate method 2 and alternate method 3, since the only difference is the use of τ_b which was shown in the results for alternate method 1 to not have a major impact on error in the range investigated.



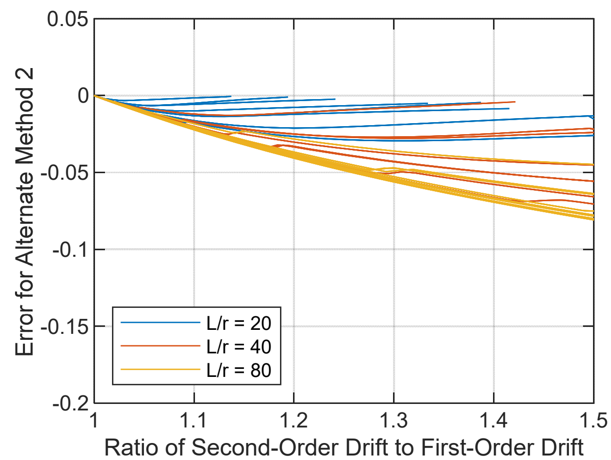
(a) Alternate Method 1



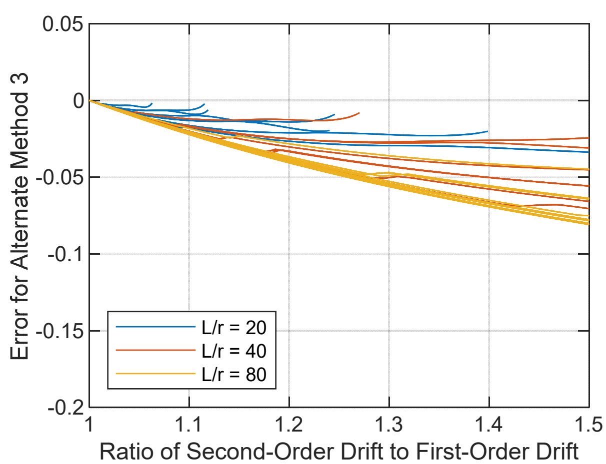
(b) Alternate Method 1
No notional load when $\Delta_2/\Delta_1 \leq 1.7$



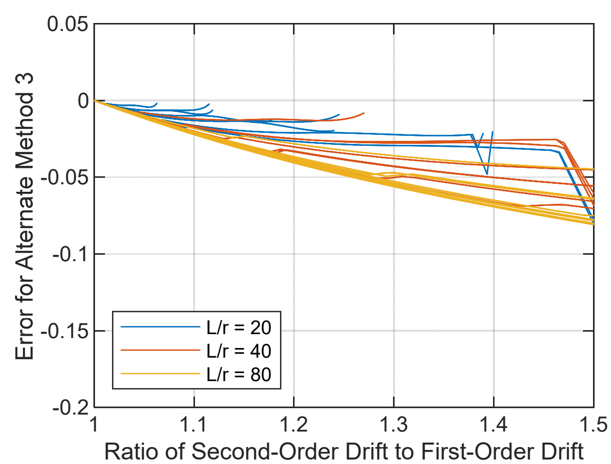
(c) Alternate Method 2



(d) Alternate Method 2
No notional load when $\Delta_2/\Delta_1 \leq 1.7$



(e) Alternate Method 3



(f) Alternate Method 3
No notional load when $\Delta_2/\Delta_1 \leq 1.7$

Figure 4: Computed error for the parametric suite of frames for alternate methods 1, 2, and 3

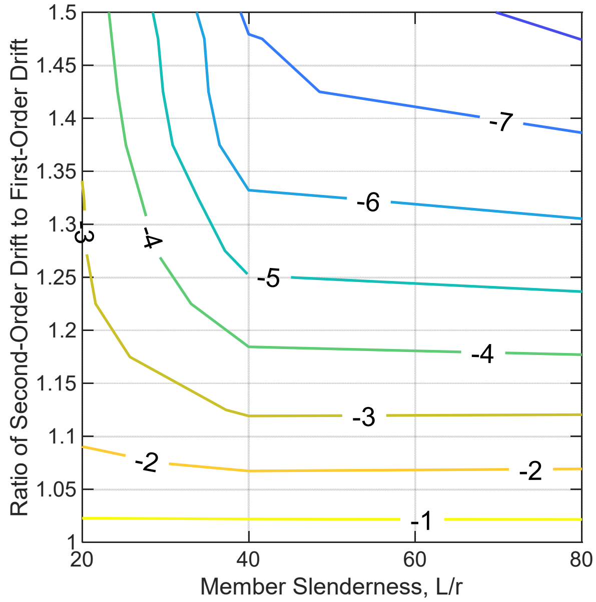


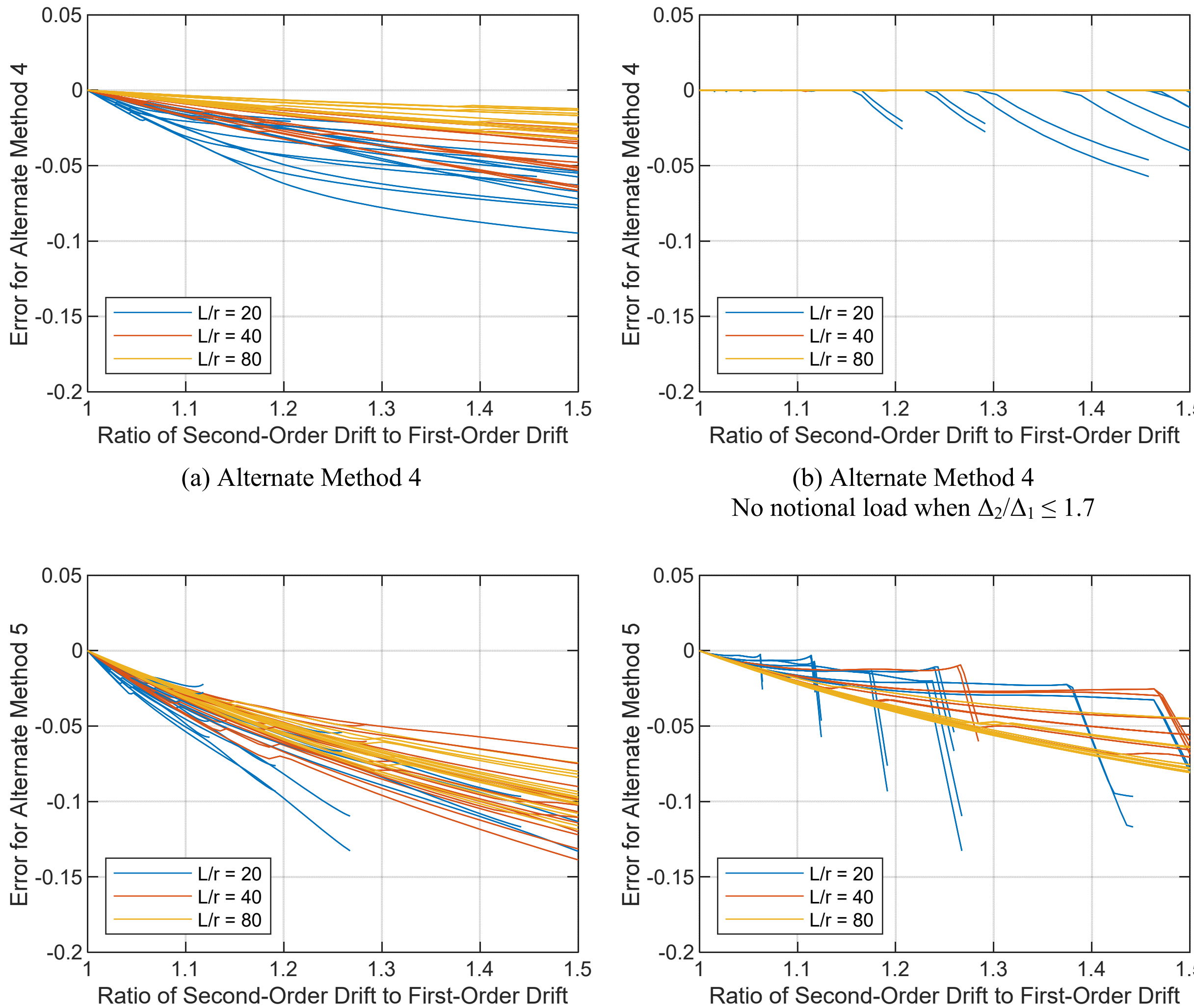
Figure 5: Contour plot showing maximum percent error for alternate method 3 across all frames investigated as a function of member slenderness and ratio of second-order drift to first-order drift. The exception to neglect notional loads when $\Delta_2/\Delta_1 \leq 1.7$ was not used for these results.

4.2 Effect of Neglecting Notional Loads

The effect of neglecting notional loads is seen by comparing alternate methods 4 and 5 to the base direct analysis method as in Fig. 6. When the exception to neglect notional load when $\Delta_2/\Delta_1 \leq 1.7$ and other lateral loads are applied is used, the effect of fully neglecting notional loads is minimal as shown in Fig. 6b. In the range investigated, error is only seen for a few of the stockiest frames and only up to a maximum of about 6%. When the exception is not used, the effect of fully neglecting notional loads is more significant as shown in Fig. 6a. The maximum error is about 3% for $\Delta_2/\Delta_1 = 1.1$ and 10% for $\Delta_2/\Delta_1 = 1.5$. The maximum error reaches 5% at about $\Delta_2/\Delta_1 = 1.15$.

Neglecting both adjustments to stiffness and notional loads as in alternate method 5 results in the greatest errors among the alternate methods, as shown in Fig. 6c and Fig. 6d. However, the errors are still relatively small when Δ_2/Δ_1 is small. The maximum error is about 6% when $\Delta_2/\Delta_1 = 1.1$.

Fig. 7 shows a contour plot of the maximum error for alternate methods 4 and 5 as a function of Δ_2/Δ_1 and L/r. The greatest errors for alternate method 3 occur for the frames with the greatest member slenderness (Fig. 5). The opposite is seen with alternate method 4, the errors are greatest for the frames with the least slender members. For alternate method 5, which combines the simplifications of alternate methods 3 and 4 by neglecting both stiffness reductions and notional loads, the error is essentially the summation of the errors from alternate methods 3 and 4. For example, at $L/r = 40$ and $\Delta_2/\Delta_1 = 1.3$, the maximum error for alternate method 3 is about -5.5% (Fig. 5). For alternate method 4, the maximum error is about -4.5% (Fig. 7a). For alternate method 5, the maximum error is about -10% (Fig. 7b).



(a) Alternate Method 4

(b) Alternate Method 4
No notional load when $\Delta_2/\Delta_1 \leq 1.7$

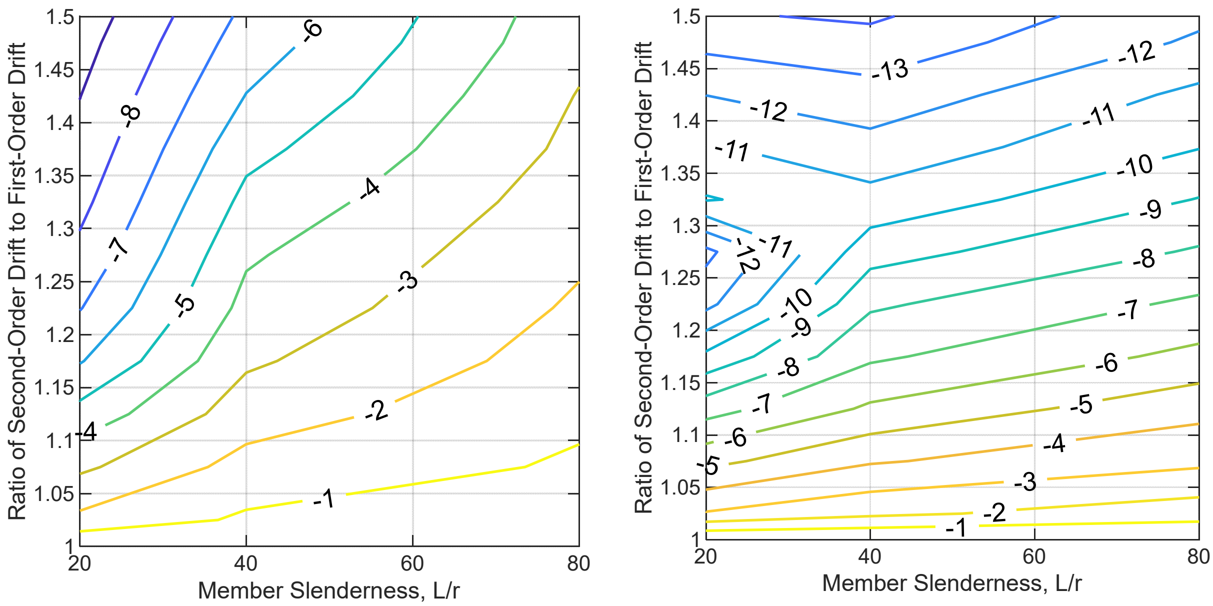
I Alternate Method 5

(d) Alternate Method 5
No notional load when $\Delta_2/\Delta_1 \leq 1.7$

Figure 6: Computed error for the parametric suite of frames for alternate methods 4 and 5

5. Conclusions

The methods of design for stability of structural steel frames presented in the AISC *Specification* (2022) are rigorous and have been developed over decades to account for the behavioral effects and parameters that impact strength. However, effects such as stiffness reductions due to inelasticity and parameters such as initial geometric imperfections do not impact all frames equally. Simplifications that streamline the design of common structures without compromising safety would be warmly received by the design community. To this end, a parametric study was performed to quantify and better understand the error that would be introduced by some potential simplifications. Error was measured as the difference in maximum permitted applied loads between alternate versions of the direct analysis method with simplifications and the base direct analysis method without simplifications. Analyses were performed on a wide range of frame configurations but only results where the ratio of second-order drift to first-order drift, Δ_2/Δ_1, was less than or equal to 1.5 were investigated. The results show that the additional stiffness reduction factor, τ_b, has little impact in the range investigated and the stiffness reduction in general has only modest impact, especially when Δ_2/Δ_1 is low. The use of notional loads also has a modest impact. The impact of neglecting stiffness reduction was greatest for the slenderest frames, and the impact of neglecting notional load was greatest for the least slender frames, however, the errors are roughly additive. As a result, neglecting both stiffness reduction and notional load (as in alternate method 5) would only be appropriate when Δ_2/Δ_1 is very low, regardless of member slenderness. While no recommendations for design can be made from these results alone, they provide the data necessary to guide further research and code development which will eventually lead to helpful and justified simplifications to the direct analysis method.



(a) Alternate Method 4 (b) Alternate Method 5

Figure 7: Contour plot showing maximum percent error across all frames investigated as a function of member slenderness and ratio of second-order drift to first-order drift. The exception to neglect notional loads when $\Delta_2/\Delta_1 \leq 1.7$ was not used in the base design method for these results.

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References

AISC. (2022). *Specification for Structural Steel Buildings*. American Institute of Steel Construction, Chicago, Illinois.

ASCE. (1997). *Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design*. American Society of Civil Engineers, Reston, Virginia.

Denavit, M. D. (2021). "Expanded Benchmark Studies on the AISC Direct Analysis Method." *Proceedings of the Annual Stability Conference*, Structural Stability Research Council.

Denavit, M. D., Hajjar, J. F., Perea, T., and Leon, R. T. (2016). "Stability Analysis and Design of Composite Structures." *Journal of Structural Engineering*, ASCE, 142(3), 04015157.

Kanchanalai, T. (1977). *The Design and Behavior of Beam-Columns in Unbraced Steel Frames*. CESRL Report No. 77-2, Structures Research Laboratory, Department of Civil Engineering, The University of Texas at Austin, Austin, Texas.

Surovek-Maleck, A. E., and White, D. W. (2004). "Alternative Approaches for Elastic Analysis and Design of Steel Frames. II: Verification Studies." *Journal of Structural Engineering*, ASCE, 130(8), 1197–1205.