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Identification of Critical Bracing Lines in Lean-on Bracing Systems and Corresponding Stiffness/Strength Expressions

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Abstract
Cross-frames are critical for the stability of steel I-girder bridges during construction and play an important role in completed bridges. Historically, cross-frame brace locations have been regions of fatigue concerns, and each brace requires significant handling and processing during fabrication. The braces represent one of the most expensive bridge components per unit weight. Therefore, there are major benefits to minimizing the number of cross-frames in a bridge in terms of economic and structural performance. Lean-on bracing concepts reduce the number of cross-frames on a bridge by replacing full cross-frames in certain bracing lines with top and bottom struts, allowing a cross-frame to brace several girders. Lean-on bracing concepts were developed for the Texas Department of Transportation (TxDOT) in the early 2000s. Previous studies developed design guidelines, but recent applications of lean-on bracing in TxDOT bridge designs demonstrated the need for improved efficiency and clarity. The stiffness and strength of a given line of bracing are functions of the number and location of cross-frames in the line, as well as the specific cross-frame geometry (X, K, or Z-shapes). While previous lean-on bracing equations were applicable to systems with one X-shaped cross-frame positioned in an exterior bay, derivations and model validation, have been completed to extend the application of the design guidance. Derived equations with simplified design expressions will be discussed in terms of stability implications, with consideration for which line(s) of bracing control based on design moments and distribution of cross-frames along the span.

1. Introduction
I-shaped girders are often utilized in steel bridge systems as an efficient and economical solution in a wide range of bridge applications. The exceptional strength-to-weight properties of steel make it a preferable material, and steel girders provide significant flexibility in terms of shipping, as the bridge girders can be fabricated in shorter lengths, shipped to the site, spliced together, and quickly

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erected. However, the high strength-to-weight ratio can lead to slender elements and systems, which may prove to be troublesome during erection and other construction phases when the bracing conditions are highly variable. During construction stages, the steel section alone generally supports the full load. Construction stages are generally critical for lateral-torsional buckling (LTB), which is a limit state that involves lateral movement of the compression flange and twist of the section, as depicted in Figure 1. Stability in the finished bridge is rarely a concern because the cured concrete deck provides continuous lateral and torsional bracing to the composite system.

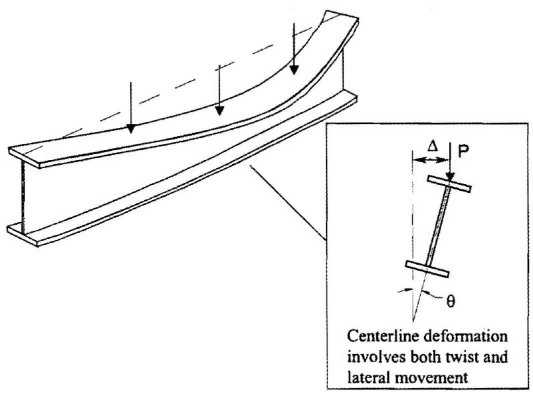

Figure 1. Lateral-Torsional Buckling. Adapted from Helwig and Wang (2003).

An increase in LTB capacity is achieved by providing adequate bracing. Effective beam bracing can be achieved by either restraining lateral displacement of the critical compression flange (lateral bracing), or by controlling the twist of the section (torsional bracing). Once the composite concrete deck has cured, the deck and shear studs provide continuous lateral and torsional restraint to the girder top flange and additional stability to the bottom flange. As a result, conventional LTB is not typically a concern in the completed bridge. As alluded to above, the critical stages for LTB are typically during erection and deck placement. In bridge I-girder systems, cross-frames and diaphragms commonly serve as stability braces during construction to enhance the LTB resistance of the girders. Since cross-frames and diaphragms restrain the twist of the cross-section at discrete locations along the length of the bridge girder, they are categorized as point (discrete) torsional braces. Though the braces are necessary for girder stability and other functions, such as restraining fascia girders from torsion applied by deck overhang brackets, they introduce some complexities into the design and require strategic placement along the length and width of the framing system. These complexities range from difficulties during fabrication and erection to concerns regarding the fatigue performance of the girder system. Due to the significant handling and fabrication requirements, the braces are often the most expensive component of steel bridges per unit weight. Therefore, it is advantageous to refine the design and detailing of cross-frame systems.

The AASHTO LRFD Bridge Design Specification (BDS) (2023) provides design, detailing, and analysis guidance for cross-frames and diaphragms, but this guidance is primarily limited to the fatigue limit state. The 9th edition of the AASHTO LRFD BDS (2020) has no formal guidance on stability bracing requirements of cross-frames and diaphragms, though a recent study that investigated the stability bracing characteristics of conventional cross-frames in steel I-girder systems resulted in recommendations that will be included in the 10th edition of the AASHTO LRFD BDS due out in 2024. However, due to the absence of formal design requirements in all current and previous editions of the AASHTO BDS, the typical practice has been to utilize standard brace details and layouts that are specified by state departments of transportation. For cross-frames in steel bridges, conventional detailing practice is to provide braces between adjacent girders across the full width of the bridge, as shown in Fig. 2 (A). However, in some applications, such a layout can lead to large live-load induced forces and difficulties with brace installation, particularly in bridges with significant support skew. Instead of providing cross-frames across the full width of the bridge, selective positioning cross-frames within the bridge cross-section and using top and bottom struts to lean other girders on the braced locations, as depicted in Fig. 2 (B), can provide improved behavior and efficiency. This concept is referred to as lean-on bracing and has long been applied to the bracing of frames for a variety of structural engineering applications. In the early 2000s, lean-on concepts were adapted for implementation into steel I-girder bridges (Helwig and Wang 2003). Lean-on braces offer a cost-effective solution by combining the versatility of a torsional bracing system with the simplicity of a lateral brace. In these systems, torsional braces (typically in the form of cross-frames) are strategically placed throughout the bridge and provide the primary source of stability to the girders.

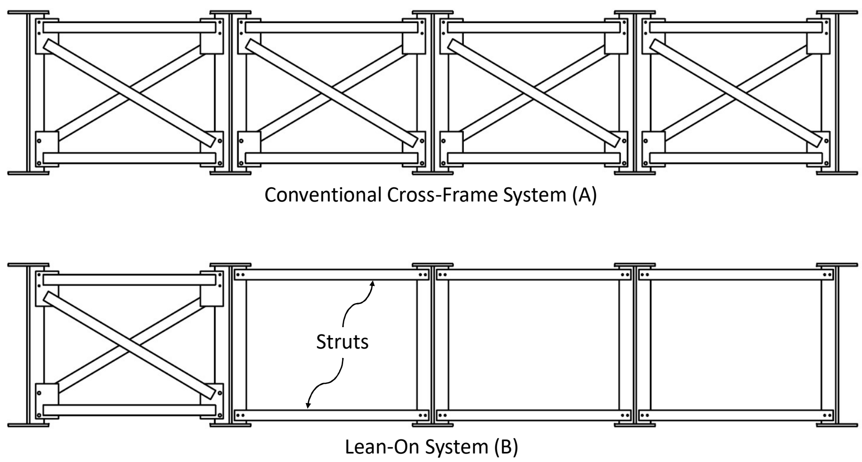

Figure 2. Conventional Cross-Frame System (A) versus a Lean-On System (B)

As noted previously, the current AASHTO LRFD provides no guidance on the design of cross-frames for stability bracing requirements. The provisions approved for inclusion in the 10th edition of AASHTO focus on the stability bracing requirements for conventional bracing. Provisions for lean-on bracing are not included in the 10th edition of the AASHTO LRFD BDS; however, there is interest in the inclusion of guidance on lean-on concepts for future editions. Based upon the recommendations from TxDOT project 0-1772 (Helwig and Wang 2003; Romage 2008) there have been successful applications of lean-on bracing in bridges with both skewed and normal supports, primarily in the state of Texas. Some of the more recent applications have identified a number of aspects of lean-on bracing that would benefit from additional research. Furthermore, there has been an abundance of research conducted over the past few decades with respect to LTB and the bracing characteristics of cross-frames, but the application towards lean-on systems was not considered. Therefore, the present research investigation was conducted to refine the design process and develop improved guidance on design procedures for wider applications of lean-on bracing.

In previous work (Helwig and Wang 2003), equations were developed for the brace stiffness of a typical bracing system and for a lean-on system with only one cross-frame in the brace line. Subsequential work (Gasser et al. 2023) introduced potential approaches for the stiffness and force distribution in lean-on systems with multiple cross-frames per line. The generalization and detailed validation of a recommended approach are discussed in this paper.

2. Background

It is necessary to begin with an understanding of the required brace stiffness of the torsional bracing system. The current equations for the provided brace stiffness of a typical bracing system are then discussed.

2.1 Brace Stiffness Requirement Equation

The recent ballot provisions for inclusion into the AASHTO bridge design specifications are generally an extension of the required brace stiffness provided in AISC (2017), and given in the following expression for full-depth cross-frames:

$$\beta_{T\,req} = \frac{2.4LM_u^2}{\varphi n E I_{eff} C_b^2} \tag{1}$$

where L is the span length, M_u is the factored design moment, φ is 0.75 (LRFD), n is the number of intermediate braces, E is the modulus of elasticity, C_b is the moment gradient factor, and I_{eff} is defined as:

$$I_{eff} = I_{yc} + \frac{t}{c} I_{yt} \tag{2}$$

where I_{yc} is the lateral moment of inertia of the compression flange, I_{yt} is the lateral moment of inertia of the tension flange, t is the distance from the centroid of the tension flange to the neutral bending axis, and c is the distance from the centroid of the compression flange to the neutral bending axis.

The expression shown in Eq. 1 approximately provides twice the ideal stiffness and is assumed to limit the twist at the brace location to a value equal to the initial imperfection, θ_0. Therefore, the resulting brace moment (M_{br}) is given by the following expression:

$$M_{br} = \beta_{T\,req'd}\theta_0 = \frac{2.4LM_u^2}{\varphi n E I_{eff} C_b^2} \frac{L_b}{500h_0} \tag{3}$$

2.2 Cross-Frame Stiffness Equation

The provided brace stiffness must meet or exceed the required brace stiffness:

$$\beta_T \geq \beta_{Treq'd} \tag{4}$$

where β_T is the total brace stiffness of the torsional system and is generally a function of three stiffness components. Most stability bracing systems follow the equations for springs in series as given by the following expression:

$$\frac{1}{\beta_T} = \frac{1}{\beta_b} + \frac{1}{\beta_g} + \frac{1}{\beta_{sec}} \tag{5}$$

where β_b is the stiffness of the brace, β_g is the in-plane girder stiffness, and β_{sec} is the stiffness of the cross-section related to cross-sectional distortion. Eq. 5 indicates that β_T is less than the smallest of the three individual stiffness components, which are assumed to interact as springs in series. From this relationship, it is evident that an otherwise stiff cross-frame can be adversely affected by poor in-plane girder stiffness or significant distortional effects in the girder webs. Thus, the overall stiffness of a torsional brace is effectively limited by the most flexible component in Eq. 5.

2.3 In-Plane Girder Stiffness, β_g

The in-plane (i.e., vertical) flexural stiffness of the bridge girders themselves contribute to the overall stiffness of the torsional bracing system. The stiffness contribution of the girders was first shown in twin-girder systems (Helwig et al. 1993). As shown in Fig. 3, when the girders are subjected to a twist, the internal moment in the cross-frame is equilibrated by vertical shear forces acting at the ends of the brace. The vertical forces on the adjacent girders cause one girder to deflect upwards and the other to deflect downwards, leading to a rigid body rotation. These deformations reduce the effectiveness of the brace. With a wider system, this displacement is reduced, as demonstrated by the four-girder system shown in the same figure.

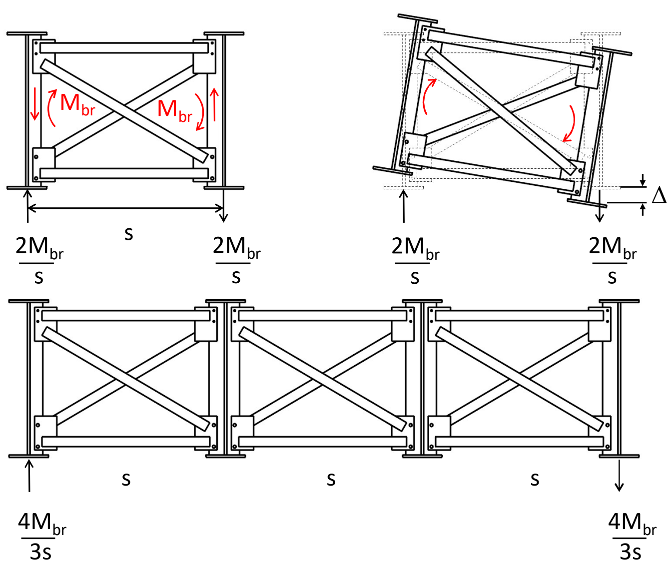

Figure 3. In-Plane Girder Stiffness.

The in-plane girder stiffness contribution is most critical in narrow systems, such as two or three-girder bridges, and is tied to a mode of buckling that is often referred to as the system buckling mode (Yura et al. 2008; Han and Helwig 2016). If β_g is less than $\beta_{T\,req'd}$, full bracing cannot be achieved regardless of the stiffness of the stiffness of the brace that is utilized. From a stability perspective, the system mode will control over buckling between the brace points. As noted previously, design guidance for the system failure mode has been incorporated into AASHTO LRFD (2023).

A revised expression for the in-plane stiffness, based on the system buckling model, has been developed and proposed by Fish (2021; 2024). Subsequent work (Gasser et al. 2024) by the research team included the development of modification factors to account for behavior due to support skew and lean-on bracing. The complete expression is given by Eq. 6.

$$\beta_g = C_{LO}^2 C_{bs}^2 \frac{\pi^4 E I_x S^2}{2n_g (KL)^3 (n+1)} \alpha_x \tag{6}$$

where C_{LO} is the lean-on layout factor, C_{bs} is the moment gradient factor, I_x is the in-plane moment of inertia of the girder, S is the girder spacing, n_g is the number of girders in the system, n is the number of brace points, K is an effective length factor used to account for warping restraint added by modifications to the bridge system, such as lateral trusses, and α_x is the system warping stiffness factor developed in Fish (2021) and shown in Table 1.

Table 1. System Warping Stiffness Factor Values

Number of Girders	System Warping Stiffness Factor
2	1
3	4
4	10
5	20
6	35
7	56
8	84
9	120
10	165

2.4 Cross-Section Stiffness, β_{sec}

Only the region outside of the brace depth contributes to the cross-sectional distortion. Because most cross-frames in bridge I-girder applications are relatively deep with respect to the girder depth, the cross-section stiffness component tends to be a large value, such that it is not usually a significant concern in Eq. 5. Language in the approved ballot for AASHTO allows β_{sec} to be taken as infinity for braces deeper than 80% of the web depth, which is relatively common in most bridges. This provision recognizes the significant stiffness for relatively deep braces. As a result, β_{sec} can often be ignored.

If the braces are relatively shallow compared to girder depth, the stiffness of the cross-section, β_{sec}, may have a significant effect. Yura and Helwig (2015) derived Eq. 7 for full-depth web stiffeners when the distance from the top cross-frame to the top of the girder is the same as the distance from the bottom of the cross-frame to the bottom of the girder. This form is included in AISC Appendix 6:

$$\beta_{sec} = \frac{3.3E}{h_w}\left(\frac{(1.5h_w)t_w^3}{12} + \frac{t_s b_s^3}{12}\right) \quad (7)$$

where h_w is the height of the web, t_w is the thickness of the web, t_s is the thickness of the stiffener, b_s is the width of the stiffener. The first term in the equation accounts for the effective moment of inertia for the part of the web assumed to participate in the distortion, and the second term accounts for the moment of inertia of the stiffener, taken about the centroid of the web.

2.5 Torsional Brace Stiffness, β_{br}

Torsional bracing systems typically utilize cross-frames or diaphragms to help bridge girders resist LTB. Cross-frames can be found in the form of X-shapes, K-shapes, and occasionally Z-shapes, as illustrated in Fig. 4. X-type braces work well with deep girders, such as in built-up I-girder bridges, while K-type braces or diaphragms are better suited for shallower girders. The torsional stiffness (i.e., the stiffness response of the brace when subjected to an in-plane moment) of the brace can be estimated based on an idealized truss model (Yura 2001; Helwig and Wang 2003).

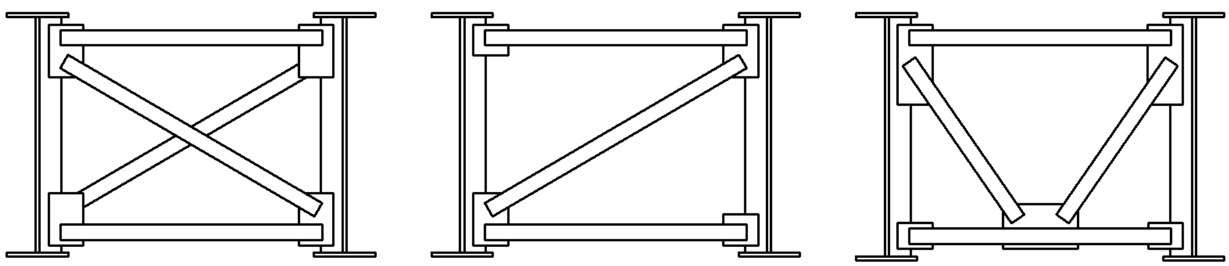

Figure 4. Various Forms of Cross-Frames (from left-to-right X-, Z-, and K-shapes).

3. Current Torsional Brace Stiffness Derivations

Two particularly relevant derivations are accepted for bracing stiffness: one for a single cross-frame, and one for a cross-frame line with a single cross-frame and lean-on struts. Both are discussed in the following sections.

3.1 Twin Girder System Derivation

Yura (2001) developed expressions for the torsional brace stiffness of cross-frames with either Z-, X-, or K-shaped geometries. The Z-shaped cross-frame is also applicable to cross-frames with two diagonals in which an engineer may conservatively neglect the compression diagonal due to the relatively low buckling strength of single-angle members that are frequently used for the braces. Such a cross-frame is often referred to as a tension-only diagonal system. The ensuing discussion focuses on the derivation for the Z-shaped system. In the derivation, the cross-frame is idealized as a truss with axially loaded members. The method of virtual work can be used to derive the expression. The idealization of this system is shown in Fig. 5. The force demand on the cross-frame consists of a force couple on both ends, as shown in the figure.

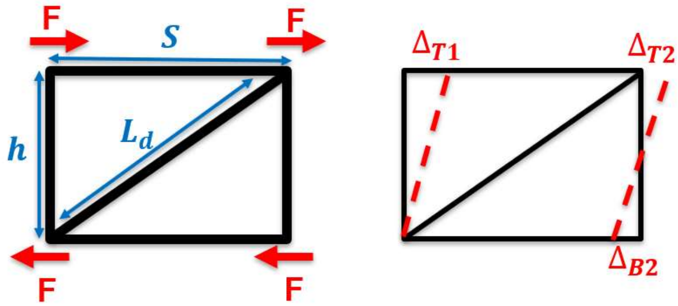

Figure 5. Twin Girder Brace Stiffness Idealization

In this approach, Eqns. 8 and 9 are combined to result in the brace stiffness (β_{br}), which is shown in Eqn. 10. The displacement of the critical girder is the basis for calculating the provided stiffness. The critical girder is the one with the largest total displacement of the top and bottom.

$$M = Fh \tag{8}$$

$$\beta_{br} = \frac{M}{\theta} \tag{9}$$

$$\beta_{br} = \frac{Fh^2}{\Delta_{crit}} \tag{10}$$

where M is the moment applied to the system, F is a unit load applied at the top and bottom of each girder in the directions shown, h is the height of the brace, θ is the rotation of the girder, and Δ_{crit} is the total displacement of the critical girder (here, $\Delta_{T2} + \Delta_{B2}$).

From the virtual work procedure, Δ_{crit} is calculated, resulting in Eqn. 11 for β_{br}. This equation represents the stiffness of a Z-shaped cross-frame. Yura (2001) presented similar derivations for X-type and K-type cross-frames.

$$\beta_{br} = \frac{h^2 S^2 E}{\frac{2L_d^3}{A_d} + \frac{S^3}{A_s}} \tag{11}$$

where h_b is the depth of the cross-frame, L_d is the length of the cross-frame diagonal members, A_d is the cross-sectional area of the cross-frame diagonals, and A_s is the cross-sectional area of the cross-frame struts.

From Eq. 11, it is evident that the torsional stiffness of the brace is a function of the axial stiffness of the individual members. Although not explicitly presented, the inherent flexibility of the connections should also be considered in the evaluation of the overall brace stiffness, similar to what is done for cross-section distortional effects or in-plane girder flexibility.

For cross-frame applications, single-angle or tee sections are often used for cross-frames and are attached to connection or gusset plates with eccentric connections to the main member. These eccentricities lead to a reduction in stiffness, as covered in Battistini et al. (2013; 2016) and Wang (2013). The stiffness reduction is accounted for with a fixed reduction factor, R. The AASHTO

LRFD (2020) recommends an R-value of 0.65 during construction and 0.75 in the completed bridge. The R-factor is applied to the cross-sectional area of the diagonals and struts in computer analyses or stiffness equations. This reduction factor was calibrated to represent the softening effects for a wide range of common cross-frame configurations, connections, and member sizes.

3.2 Lean-On Bracing Derivation 2003

Similar to the stiffness of a single cross-frame developed by Yura (2001), Helwig and Wang (2003) derived a generalized equation for a Z-shaped cross-frame or X-shaped cross frame with the tension only assumption. The brace stiffness contribution in a lean-on bracing system that reflected the bracing load path of a series of adjacent girders restrained by top and bottom struts with a single cross-frame at one end of the bracing line (exterior alignment) is given by Eqn. 12.

$$\beta_{br,2003} = \frac{h^2 S^2 E}{\frac{n_{gc} L_d^3}{A_d} + \frac{S^3}{A_s}\left(n_{gc}-1\right)^2} \tag{12}$$

where n_{gc} is the number of girders per cross-frame.

Based on the specific geometry that was considered in the derivation, the number of cross-frames per bracing line is assumed to be one, so n_{gc} is effectively the number of girders. The method of virtual work can be used to account for the axial shortening of the struts and diagonals based on the respective forces in the individual members. As an example, the idealization of a four-girder system is shown in Fig. 6. The free-body diagram shows the accumulation of forces that develop across the width of the bridge. The bracing demand from the girders results in force couples that lead to the forces indicated in the figure.

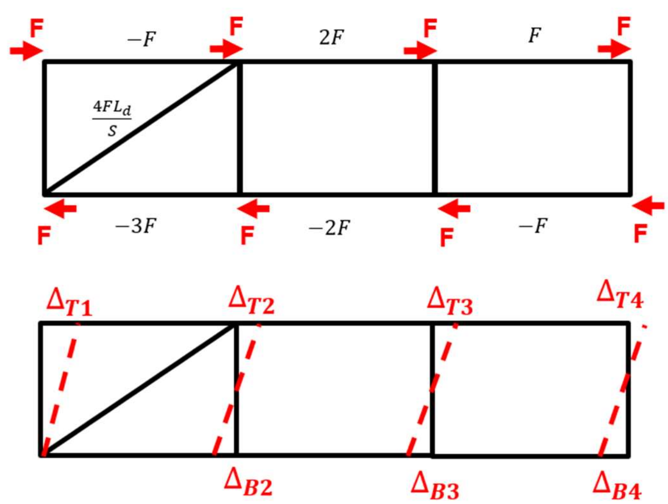

Figure 6. Lean-On Bracing Stiffness Idealization

The expression in Eq. 12 has been used in some designs for lean-on systems where more than one cross-frame in a given bracing line. Due to the definition for n_{gc} in Helwig and Wang (2003), which was the number of girders per cross-frame, designers simply divided the number of girders by the number of cross-frames. Although such an interpretation makes sense from the definition of the variable – the expression was not derived for such an application.

4. Approach for Multiple Cross-Frames

Because the lean-on bracing stiffness equation given by Eq. 12 is only applicable to cross-frame lines with one Z-frame or tension model X-frame in an exterior bay, it is advantageous to study ways to modify the expression to account for varied number, location, and type of cross-frame. Gasser et al. (2023) introduced potential procedures to accommodate for the presence of multiple adjacent cross-frames aligned to one side of a bracing line. The following sections review the recommended approach and introduce a generalized brace stiffness equation.

4.1. Description of Cross-Section Slice Approach

Based on a comparison of several approaches and initial results (Gasser et al. 2023), the recommended approach for accounting for lean-on brace lines with multiple cross-frames is the Cross-Section Slice (CSS). In this idealization, a redundant cross-frame is essentially ignored. This is shown for a four-girder system with two cross-frames in Fig. 7, where the left cross-frame is not considered in determining the brace stiffness. This results in Δ_{crit} equal to the sum of $\Delta_{T3}, \Delta_{T4}, \Delta_{B3}, and\ \Delta_{B4.}$ The brace stiffness of the configuration is then calculated using Eq. 12 for virtual work for a three-girder system with one exterior cross-frame, resulting in Eq. 13.

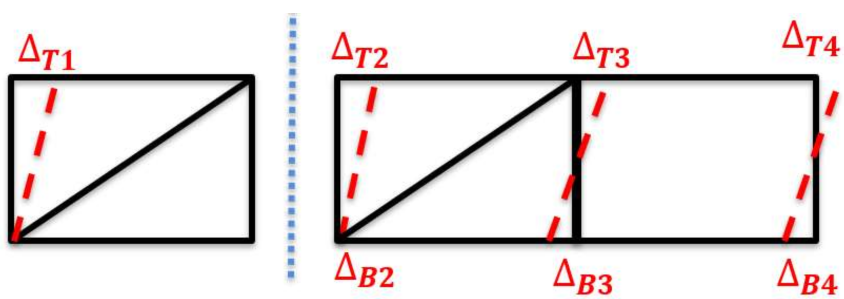

Figure 7. Cross-Section Slice Idealization

$$\beta_{br,3G} = \frac{Fh^2}{\Delta_{crit}} = \frac{Fh^2}{\frac{3FL_d^3}{S^2 A_d E} + \frac{4FS}{A_S E}} \tag{13}$$

4.2 Generalization of the Cross-Section Slice Approach

In order to construct a generally applicable equation, the CSS approach was applied to Eq. 12, to result in Eq. 14. The n_{gc} term was substituted for separate terms representing the number of girders and the number of cross-frames separately.

$$\beta_{br,css} = \frac{ES^2h^2}{\frac{(n_g - n_c + 1)L_d^3}{A_d} + \frac{(n_g - n_c)^2 S^3}{A_s}} \tag{14}$$

where n_g is the number of girders and n_c is the number of cross-frames in the bracing line.

5. Detailed Validation

A finite element model was used to validate the derived generalized CSS equation for varying cross-frame line configurations. The model validation process and results are described in the following sections.

5.1 Model Validation

In order to validate the derived CSS expression, it was necessary to first develop models of cross-frame system sections with the same assumptions as previous derivations (J. A. Yura 2001; T. A. Helwig and Wang 2003). SAP2000 was the finite element program selected. The bracing system was idealized as a two-dimensional truss system. All members were moment released at both ends. The girders were assumed to have infinite stiffness (which was later shown to be inconsequential), and the cross-frame line was simply supported with lateral unit loads applied at the top and bottom of each girder, as shown in Fig. 8.

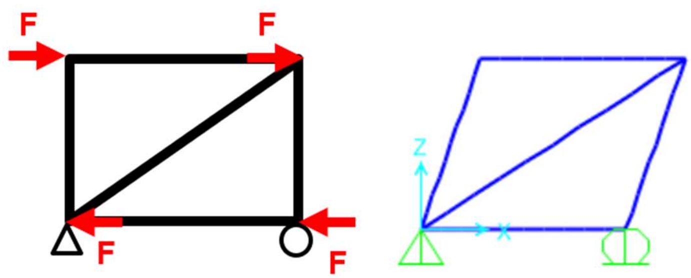

Figure 8. Twin Girder Tension Model Idealization and Deformed Shape

In order to validate the modeling procedure for an analysis of the brace stiffness (β_{br}), the SAP models were compared with analytical solutions. First, a twin-girder system was considered. The current equation for the brace stiffness based on the shown tension model is given by Eq. 11 (J. A. Yura 2001). The model was also run with both cross-frame diagonals in order to quantify the level of conservatism of the tension system equation. To calculate the stiffness of the model, Eq. 10 was used, with Δ_{crit} indicating the sum of the respective displacements at the top and bottom of the right girder. Negligible error (less than 1.0%) resulted between the tension system SAP model and the tension system equation, which indicates the model is performing as expected. Additionally, it was found that the full cross-frame SAP model has more than double the stiffness, indicating that the addition of the second cross-frame diagonal significantly impacts the stiffness of the brace.

5.2 Detailed Validation of Generalized Equation

The results from the generalized CSS equation (Eq. 14) were compared with finite element analysis results for varied cross-frame layouts. The stiffness obtained from the CSS equation was divided by the stiffness obtained from the SAP model to obtain a ratio where a value greater than one indicated the CSS equation was unconservative. Values less than or equal to one indicate an accurate or conservative expression. In all configurations, the equation was found to be exact or conservative. The analysis sets are discussed in detail below for a modulus of 29,000 ksi, an area of 6.45 in^2 for all members, girder spacing of 96 inches, and cross-frame depth of 76 inches.

The exterior cross-frame position aligned to the left was studied first. One to nine cross-frames were positioned next to each other on one side of two to ten girder systems. The stiffnesses resulting from each of the three calculations were normalized against the model stiffness, such that a value greater than 1.0 would indicate the equation giving a larger stiffness than the model, which is unconservative. In this configuration, the CSS approach was always conservative. For layouts with exactly one cross-frame, the equation predicted the exact same stiffness as the model. For cross-frame lines with more cross-frames, the conservatism of the equation relative to the model increased to a maximum of 25%. The results are depicted in Fig. 9. Example cross-sections for six-girder bridges are shown to the left of the plot. Note that the right plot includes results for layouts with two to ten girders.

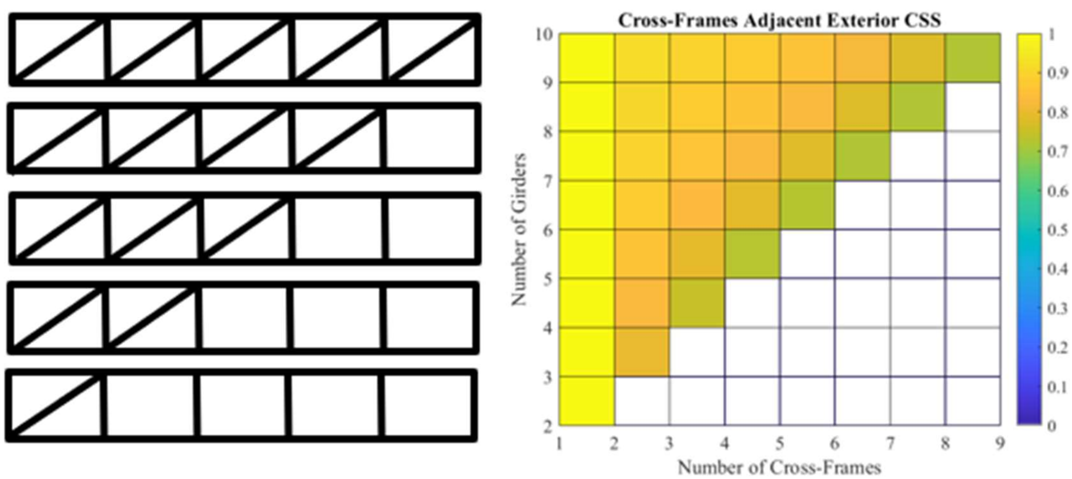

Figure 9. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for Exterior Z-Frames

Next, a similar approach was applied to a checkerboard pattern, with the cross-frame line bays alternating cross-frames and lean-on struts. In this layout, the least conservative configuration is an even number of girders missing cross-frames on ends. However, the equation was conservative for all layouts. The equation became increasingly more conservative with more girders, as shown in Fig. 10Figure 10. The left side of the figure illustrates six-girder layouts, and the right plot provides the result for layouts with two to eleven girders.

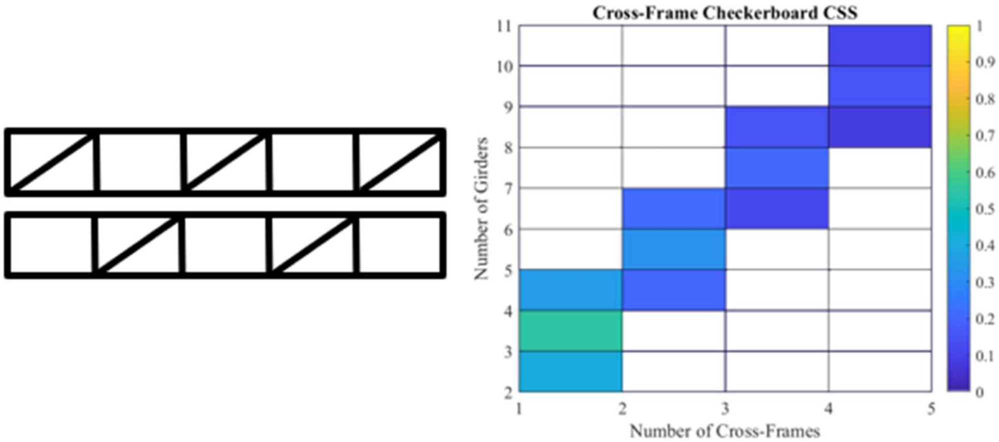

Figure 10. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for Checkerboard Z-Frames

Cross-frame placements for interior and exterior bays were compared, as depicted in Fig. 11. Interestingly, the equation was most conservative for layouts with cross-frames placed in the two exterior bays, even compared to layouts with more cross-frames placed in interior bays.

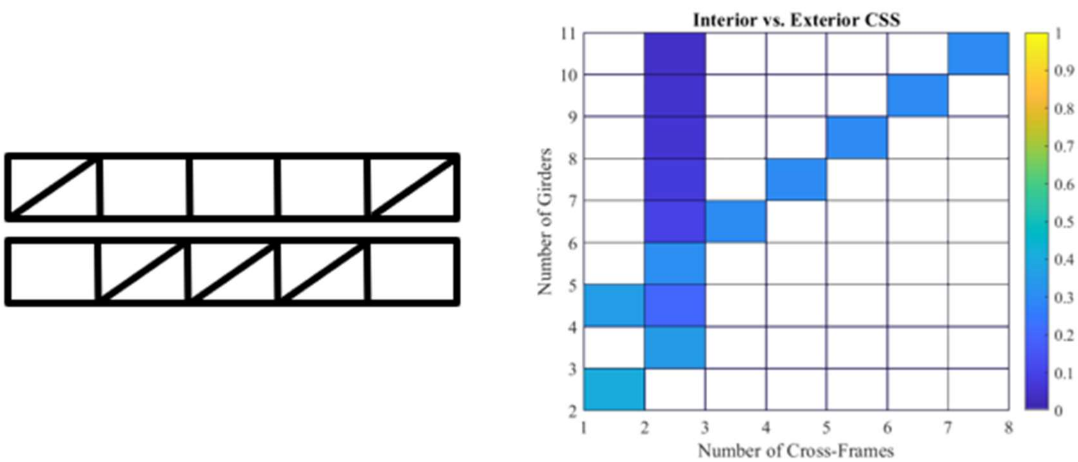

Figure 11. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for Interior vs. Exterior Z-Frames

In the next set of models, the number of cross-frames was kept constant, but the position of the cross-frame was changed. In the charts, the cross-frame axis was changed from the number of cross-frames to the position (or bay) of the first cross-frame in the line. In the case of the single cross-frame, all three equations resulted in the same values because they all reduce to the same equation for one cross-frame. These results are shown in Figs. 12, 13, and 14. The equation was more conservative for cross-frames placed in interior bays, and most conservative for cross-frames placed furthest from the pin support. This confirms that the limiting cross-section pattern is cross-frames aligned to one side.

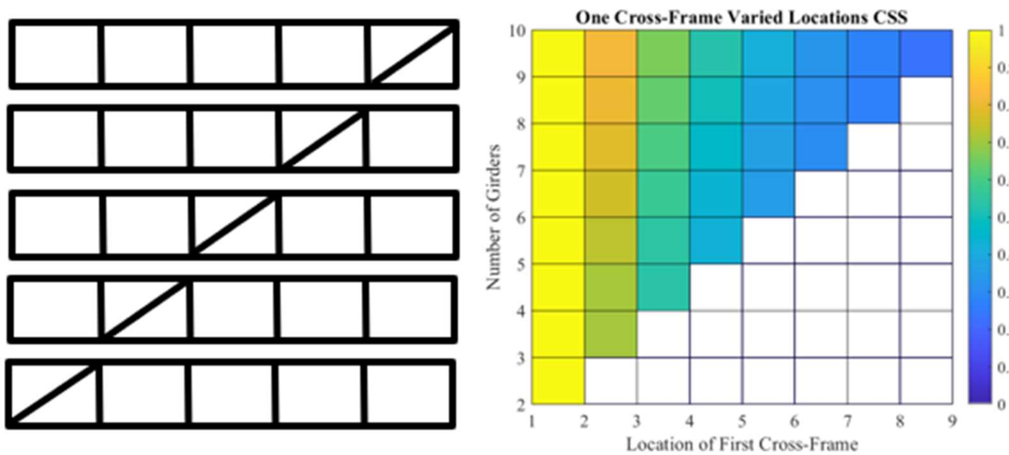

Figure 12. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for One Z-Frame Varied Location

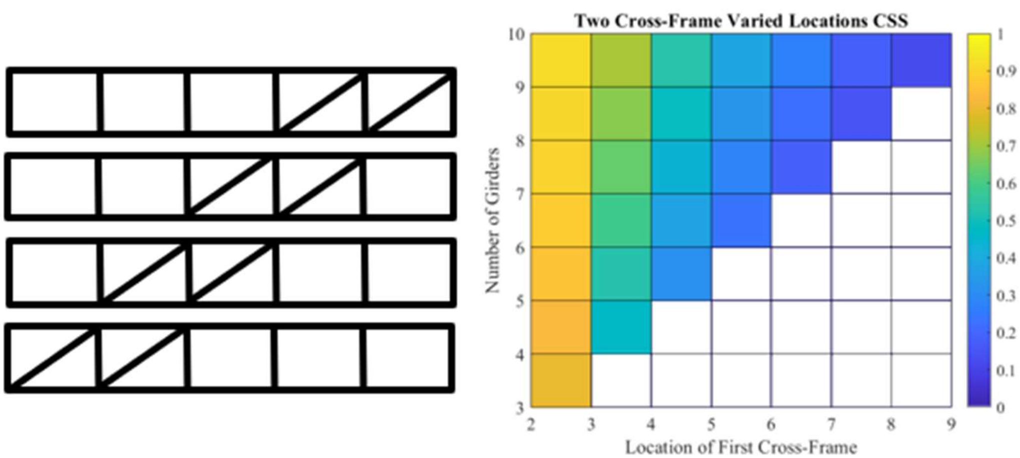

Figure 13. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for Two Z-Frames Varied Location

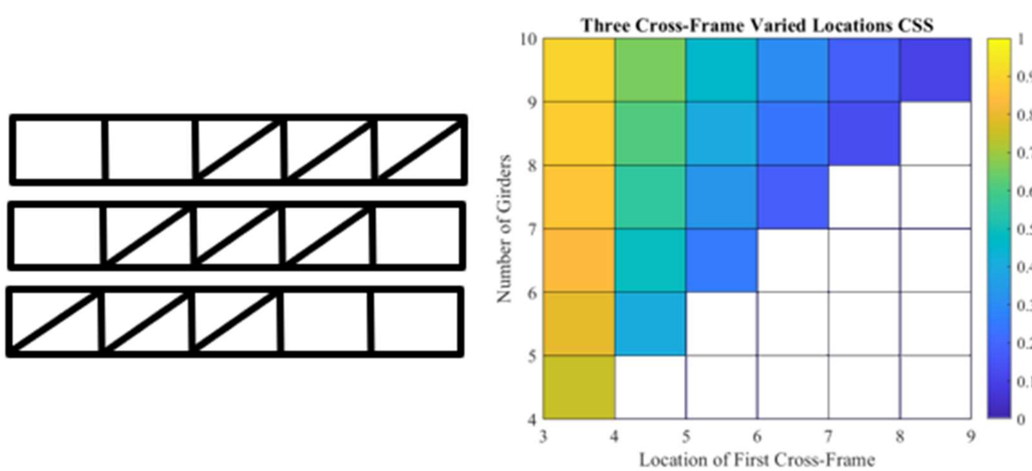

Figure 14. $\frac{\beta_{br,CSS}}{\beta_{br,SAP\ model}}$ for Z-Frames Three Cross-Frames Varied Location

Overall, the CSS equation results in perfect agreement with the SAP model for systems with only one exterior cross-frame and becomes increasingly conservative for systems approaching conventional bracing. The CSS approach results in the same stiffness value regardless of cross-frame placement within the cross-frame line, as it is only dependent on the number of cross-frames and girders. The limiting cross-frame placement was found to be an exterior cross-frame alignment, which is where all of the cross-frames were placed in adjacent exterior bays (Fig. 9Figure 9). This results in the largest possible number of adjacent lean-on bays.

6. Conclusion and Ongoing Work

The CSS approach introduced by Gasser (2023) was generalized and validated against models of hundreds of cross-frame line configurations. The equation matches the model stiffness exactly for layouts with one exterior cross-frame and is conservative for all cross-frame line layouts. Work is ongoing to modify the expression to account for nonadjacent and interior cross-frame placements more precisely. Derivations to account for the contribution of the compression diagonal in X-frames and geometry of K-frames are in progress. Additionally, studies are ongoing to assess the total system stiffness due to the varied brace stiffness along the span of lean-on systems and determine ideal cross-frame layouts.

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