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Bending and shear buckling interaction resistance of hybrid steel girders

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Abstract
Hybrid girders combining normal strength steel in the web and high strength steel in the flanges gives an economical solution for steel buildings and bridges. The bending and shear buckling resistance of conventional steel girders is deeply researched in the past and reliable resistance models are available for appropriate design. However, the behaviour of hybrid girders is not a widely researched topic. Therefore, current research aims to investigate the bending (M), shear buckling (V), and the M-V interaction resistance of hybrid steel girders. If high strength steel material is applied in the flanges and normal strength steel in the web, partial plastic stress redistribution can occur in the web by reaching the yield strength in the flange. The current study investigates, how these plastic strains influence the structural behaviour, the bending moment, the shear buckling and the M-V interaction resistances. Within the current research program numerical investigations are performed on welded box-section and I-section beams. Within the current paper investigations on I-section beams are introduced. The specialties of the structural behavior of hybrid girders are studied and evaluated. The calculation method of the bending moment resistance is widely investigated and resistance models using elastic and plastic design are introduced. Regarding the shear buckling resistance, significant investigations are executed to investigate the web and flange contribution in hybrid girders. Improved resistance model is developed considering the yield strength ratio between the web and flange plates. Finally, the M-V interaction behaviour is studied on hybrid girders and the applicability of the previously developed M-V interaction equations are evaluated.

1. Introduction
The application of hybrid constructions is rising nowadays, including steel-concrete, concrete-high strength concrete, concrete-plastics (composites), and steel-high performance steel structures. The largest advantage of hybrid girders is that each material could be used on their most effective position to take benefits from their larger strength. In the case of conventional steel I-girders, bending moment is mainly carried by the flanges and shear force by the web. Thus, bending moment is usually significantly more dominant in conventional steel I-girders, it makes sense to increase the steel grade of the flanges, and keeping the same steel grade for the web, which can still carry the shear force with adequate safety. Therefore, the application of hybrid steel girders

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using high strength steel (HSS - yield strength greater than or equal to 460 MPa) for the flanges and normal strength steel (NSS - S235 or S355) for the web gives an economical solution. Compared with normal strength steels (NSS), the use of HSS enables the selection of smaller cross-section sizes for structural components, resulting in reductions in weight, as well as enabling more streamlined and elegant structures, as highlighted by Zhu et al. (2023). Additionally, hybrid girders offer a more sustainable solution with the potential for significant reductions in CO_2 emissions, aligning with the European Union's 2030 plan for achieving climate neutrality by 2050 as highlighted by Terreros-Bedoya (2023). Therefore, hybridization, particularly application of steel-steel hybrid girders provides a promising solution to address the challenges faced by the today’s construction industry.

In the last decades application of high strength steel structures gained increasing use in the structural engineering industry as shown by Bjorhovde (2004), Miki et al. (2002) or Graham (2006) and by many other researchers. However, at the early stage, there were no design provisions to apply these structures in the civil engineering praxis. Previously, the Eurocode 3 EN 1993-1-12 design code gave additional design rules for design of high strength steel structures up to the steel grade of S700 (yield strength of ~100 kips). Recently, many experimental, numerical and analytical investigations were performed to investigate the structural behaviour and design specialties of high strength steel structures. An early and wide range summary was written by Eleni Gogou (2012) or later by Li et al. (2020). Large number of research papers also proved the benefits of high strength steel structures and gives guidelines for their design. Based on the extensive research work, the Eurocode extended its application range, and the general rules given in EN 1993-1-1:2024 will be applicable up to steel grades of S700 in the second-generation Eurocodes. In parallel the EN 1993-1-12 will be transformed to cover design rules using steel grades stronger than S700 and up to S960 (~140 kips). It means, research activities and their results are implemented into design codes and their widespread application will be possible soon.

However, application of hybrid girders, mixing NSS and HSS material gives new tasks for researchers and code developers, because the structural behaviour of hybrid girders can differ from the pure NSS and HSS structures. For example, in the case of bending resistance, if plastic design is used, the web can suffer significantly larger plastic strains by reaching the plastic resistance of the entire cross-section compared to non-hybrid girders. It is to be checked if the material can tolerate these plastic strains without premature fracture. Therefore, the plastic resistance calculation method of hybrid girders should be carefully investigated. Similar investigations should be also made for the elastic moment capacity calculation. In this case, before reaching the yield strength of the flange, the web can suffer partial plastification, which considerability in the design is to be checked. The application of full elastic design would be overconservative and uneconomical for hybrid girders. These questions were mainly addressed and answered by the research work of Zhu et al. (2023). The authors came to final conclusion for the bending resistance calculation of high strength steel and hybrid girders using HSS in the flanges. However, similar calculations are still missing from the international literature for the web buckling resistance calculation for class 4 cross-section (based on the European cross-section classification scheme – class 1-2 cross sections are design by plastic resistance; class 3 cross-section by elastic resistance and class 4 sections are designed for plate buckling). Within the current paper class 1-3 cross-sections will be studied, the analysis of class 4 cross-section will be the topic of future research papers.

Another important question is how the flange contribution in the shear buckling resistance changes if web is kept normal strength steel and the flanges are changed to high strength steel. The flange contribution in the shear buckling resistance is usually negligible for building-type structures, but it can have a significant effect for bridge-type structures, where the effect of the heavy high strength steel flanges can play an important role in the design process. The shear buckling resistance contains two parts: (i) web contribution and (ii) flange contribution. The design equation of the web contribution contains the yield strength of the web, which remains unchanged for the hybrid girders. However, the yield strength of the flange is non-linearly considered in the flange contribution resistance calculation which applicability is questionable for hybrid girders.

The third question is how hybrid girders could be used and designed for bending moment (M) and shear force (V) interaction. If plastic stress distribution is allowed in the web of the cross-section within the bending moment resistance calculation, it can result in a resistance reduction in the M-V interaction behavior. The current M-V interaction resistance models are developed for NSS structures, which accuracy and applicability should be checked for hybrid girders.

The current paper tries to find answers to the above given questions and shows the typical structural behaviour of hybrid girders subjected to pure bending moment, shear force and M-V interaction. Within the paper, at first the previous investigation on the hybrid girders is introduced. For those failure modes, where there are no relevant investigations, the research results regarding NSS or HSS will be presented. The details of the applied numerical models will be introduced which will be used for the investigation of the structural behaviour and resistance of hybrid girders. Finally, the numerical results are presented separately for bending, shear buckling and the M-V interaction behaviour. Limitations and errors of the Eurocode-based design method for hybrid girders will be highlighted and possible improvements are proposed. The used notations within the paper are shown in Fig. 1.

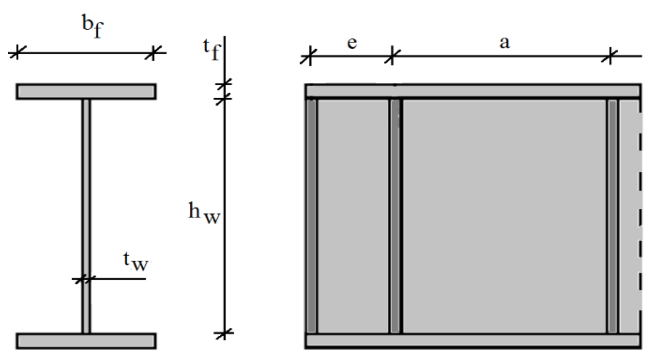


Figure 1: Used notations for plated girder geometry.

2. Previous investigations and design rules for hybrid girders

2.1 Previous investigations of bending resistance

Regarding the bending resistance of hybrid girder one of the earliest investigations is made by Veljkovic and Johansson (2004). This publication states that hybrid girders have been used in the US since long, but they are not commonly used in Europe. However, hybrid girders are more economical than homogenous girders. Veljkovic stated that the bending resistance is influenced by the difference in yield strength between flanges and webs. The web will be partially yielded before the flanges reach their yield strength. The influence of this fact will be different for different cross-section classes. Therefore, different design models are proposed for different cross-section classes, as shown in Fig. 2. For cross-section classes 1-2, the bending resistance is calculated from a fully yielded cross-section as shown in Fig. 2a. In the case of cross-section class 3, the bending resistance can be calculated from the assumption, that the flanges can reach their yield strength, the web comes in partial plastic situation which plasticism can be considered in the bending moment calculation. In the case of class 4 cross-sections, where the web is sensitive for plate buckling, the stress distribution shown in Fig. 2b) can be considered, and the bending resistance should be calculated by considering partial plastic stress distribution within the web – even if it buckles and the bending resistance can be calculated from the assumption, that the flanges can reach their yield strength. Following EN1993-1-5 design rules, the elastic section modulus W should be calculated at the mid-plane of the flanges.

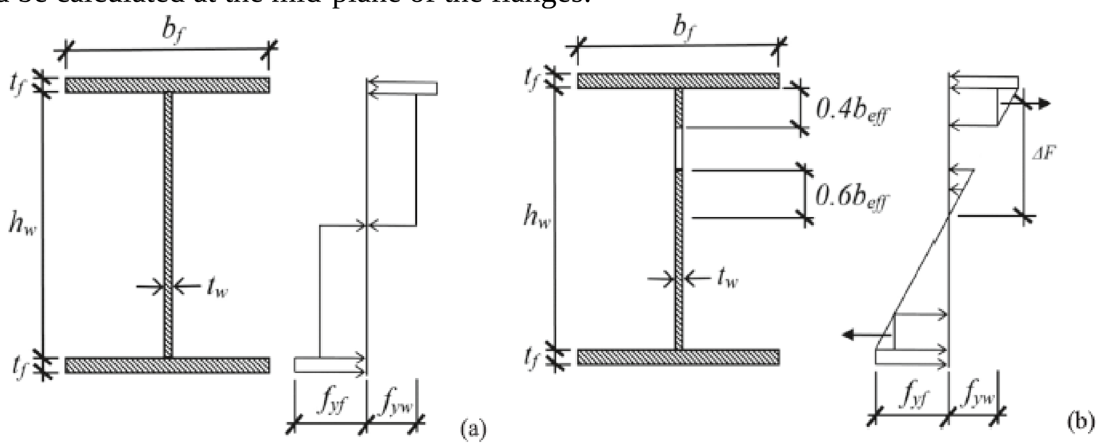


Figure 2: Bending resistance calculation model proposed by Veljkovic and Johansson (2004): a) class 1-2 cross-sections – plastic design; b) class 4 cross-sections – elastic / partially plastic design.

Another investigation in this topic made by Lateef et al. (2019) recommended the limit value of h_w/t_w=120 for class 4 cross-section, with which the cost-benefit ratio is well balanced. Another aspect on which the different investigations agree is that the bending capacity is significantly affected by the amount and distance between the flanges' lateral bracings and the web stiffeners. These bracings should limit the occurrence of local or torsional buckling failures in the elements, as expressed in the paper written by Wang et al. (2016).

The bending resistance of class 1-3 cross-section beams were extensively analyzed by Zhu et al. (2023) experimentally and numerically. Their experimental research program consisted of 4 HSS specimens having S690 steel grades, and 2 hybrid girders having S690 flanges and S355 web. The experimental investigations were extended by numerical simulations performing a large numerical parametric study to investigate the bending resistance of HSS and hybrid girders separately. Based on their investigations it was confirmed that the current slenderness limits for class 2 and 3 cross-sections according to EN1993-1-1 are suitable to classify the outstand flanges (in compression) and internal webs (in bending) of both HSS homogeneous and hybrid welded I-sections. To satisfy the rotation capacity requirement of the Eurocode design principles, stricter Class 1 slenderness limits for outstand flanges in compression (i.e. $b_f/t_f\varepsilon_f = 8$) and internal webs in bending ($h_w/t_w\varepsilon_w = 60$) were proposed. Furthermore, both the experimental and numerical results proved the accuracy of the above presented bending resistance calculation methods for beams having class 1-3 cross-sections. Detailed analysis of class 4 cross-sections is still missing from the international literature.

2.2 Previous investigations of shear buckling and interaction resistance

To calculate shear buckling resistance, EN1993-1-5 provides formulas that already account for the flanges and web yield strengths. Furthermore Terreros-Bedoya et al. (2023) pointed out that for the class 3 and 4 sections, the Eurocode design rules for the interaction between shear and bending can be applied without modification. However, for the class 1 and 2 sections, there is a debate regarding whether to neglect this interaction, despite suggestions to the contrary. Thus, in the current paper the shear buckling resistance model according to EN1993-1-5 will be checked, this design approach is presented. The bases of the shear buckling resistance model given by EN 1993-1-5 is the so called "rotated stress field method" originally developed by Höglund (1997). This design method was originally developed for unstiffened webs, and it was later extended for panels with longitudinal stiffeners. According to this model, the shear buckling resistance can be determined from the sum of the contribution of the web ($V_{bw,Rd}$) and flange ($V_{bf,Rd}$) resistances according to *Eq. (1)*.

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \leq \frac{\eta \cdot f_y \cdot h_w \cdot t_w}{\sqrt{3} \cdot \gamma_{M1}} \tag{1}$$

where $V_{bf,Rd}$ should be reduced if the girder is subjected to an accompanying bending moment. h_w and t_w are the web depth and thickness, respectively, γ_{M1} is the partial safety factor for stability checks, and η is modification factor, which is proposed to be equal to 1.2 for $f_y \leq 460$ and 1.0 otherwise. The contribution of the web and the flange in the shear buckling resistance can be calculated by *Eqs. (2)–(3)*.

$$V_{bw,Rd} = \frac{\chi_w \cdot f_y \cdot h_w \cdot t_w}{\sqrt{3} \cdot \gamma_{M1}} \tag{2}$$

$$V_{bf,Rd} = \frac{b_f \cdot t_f^2 \cdot f_{yf}}{c \cdot \gamma_{M1}} \left(1 - \left(\frac{M}{M_{f,Rd}}\right)^2\right) \tag{3}$$

$$c = a \cdot \left(0.25 + \frac{1.6 \cdot b_f \cdot {t_f}^2 \cdot f_{yf}}{t_w \cdot {h_w}^2 \cdot f_{yw}}\right) \tag{4}$$

where χ_w is the reduction factor for shear buckling, b_f and t_f are the flange width and thickness, respectively, c is the distance between the transverse stiffener and plastic hinge developed in the flange, as given by *Eq. (4)*, a is the distance between the transverse stiffeners, M is the bending moment acting in the analysed cross-section, and $M_{f,Rd}$ is the bending moment resistance of the flanges alone. The reduction factor should be calculated based on the slenderness of the web panel. For plates with rigid transverse stiffeners the shear buckling coefficient can be obtained by *Eqs. (5a)–(5b)*.

$$k_\tau = 5.34 + \frac{4.0}{\alpha^2} \qquad \text{if } \alpha \geq 1.0 \tag{5a}$$

$$k_\tau = 4.0 + \frac{5.34}{\alpha^2} \qquad \text{if } \alpha < 1.0 \tag{5b}$$

where α is the aspect ratio of the web panel (a/h_w). Further background information on the resistance model can be found in the JRC publication written by Johansson et al. (2007).

Recently, numerous research activities tested the accuracy of this design model. Jáger et al. (2017) investigated it for unstiffened and longitudinally stiffened girders having slender webs. It was also studied by Pavlovčič et al. in 2007 using laboratory tests and FE simulations. The two latest investigations in this topic, which gave similar results are made by Jáger et al. (2019) and Pedro et al. (2022). Based on their investigations the calculation method of the parameter c has been changed. The latest and most accurate proposal is given by Pedro et al. given by Eq. (6). The physical meaning of the modified parameter c is shown in Fig. 3.

$$\text{best fit proposal:} \qquad c = a \cdot 1.6 \cdot \left(\frac{b_f \cdot t_f{}^2 \cdot f_{yf}}{t_w \cdot h_w{}^2 \cdot f_{yw}}\right)^{0.44} \tag{6}$$

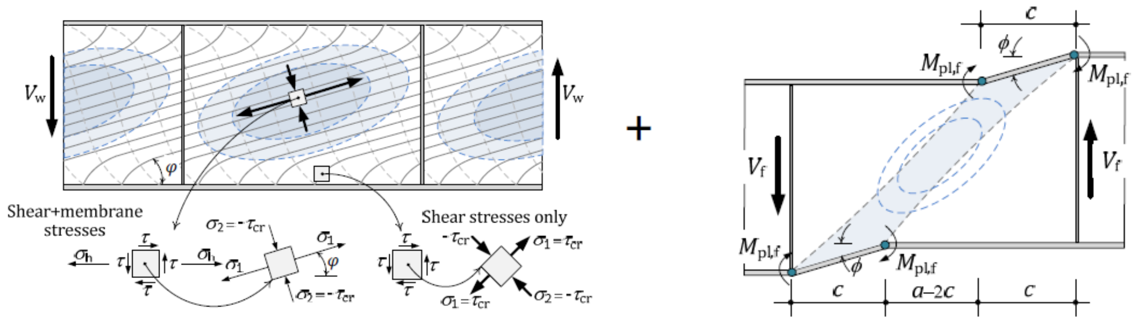


Figure 3: Höglund's shear buckling resistance model according to Pedro and Nascimento (2022).

Regarding the M-V interaction behaviour, the latest proposal, also implemented into the second-generation EN 1993-1-5 was developed by Jáger et al. (2019). The interaction equations are given by Eqs. (7)-(8).

$$\frac{M}{M_{el,eff,R}} + \left(1 - \frac{M_{f,R}}{M_{el,eff,R}}\right)\left(\frac{2 \cdot V}{V_{bw,R}} - 1\right)^{\kappa} \leq 1.0 \quad \text{if} \quad 0.5 \cdot V_{bw,R} \leq V < V_{bw,R}\,, \tag{7}$$

$$\kappa = \left(\frac{M_{f,R}}{M_{el,eff,R}} + 0.2\right)^{15} + 1 \tag{8}$$

where $M_{f,R}$ is the moment of resistance of the cross-section considering the effective area of the flanges alone, $M_{el,eff,Rd}$ is the elastic or effective moment of resistance of the cross-section depending on its cross-section class, $V_{bw,R}$ is the shear buckling resistance of the web panel alone, M and V are the applied bending moment and shear force. The M-V interaction resistance models and the M-V interaction equations have been developed for NSS girders and never checked for HSS or hybrid girders.

It should be observed, that Eq. (7) defines the M-V interaction resistance, if shear force acting on the girder is smaller than the web contribution within the shear buckling resistance, or in the opposite direction, if the applied bending moment is larger than the bending moment load carrying capacity of the flanges alone ($M_{f,R}$). Equation (1) also contains an M-V interaction resistance; therefore, the flange contribution depends on the applied bending moment. It has its largest value by M_{Ed}=0 and its smallest one by $M_{Ed} = M_{f,R}$, when Eq. (3) leads to zero addition to the shear buckling resistance. Therefore, if the accuracy of the M-V interaction resistance model of the hybrid girders is to be checked, the accuracy of both equations should be analyzed and evaluated. It will be made in the current paper.

3. Numerical modeling

3.1 Numerical model development

The numerical model is developed in the general-purpose finite element software ANSYS 19.0. The ultimate resistance is determined by geometrical and material nonlinear analysis using equivalent geometric imperfections (GMNIA). Full Newton-Raphson approach is used in the nonlinear analysis with 0.1% convergence tolerance of the residual force based Euclidian norm. Figure 4 shows the geometrical model with boundary and loading conditions used. The applied model is a full shell model using four-node thin shell elements (SHELL181). At the load introduction location (left end of the model), two transverse stiffeners are applied forming rigid end-post layout, which is important from the shear buckling resistance calculation point of view.

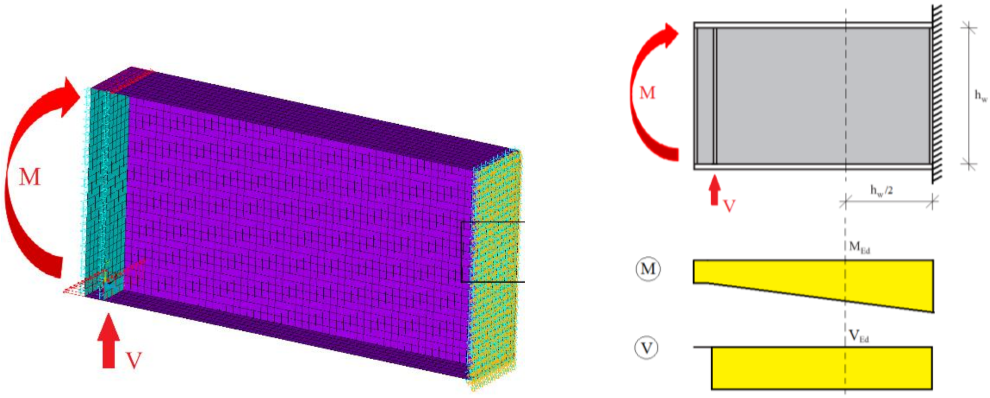


Figure 4: FE model and applied boundary and loading conditions.

At the right end of the girder, symmetry boundary conditions are applied – nodes are constrained against rotation around the longitudinal and transversal axes and against longitudinal and vertical displacements. The bending moment is applied by force couple acting at the location of the flange middle lines. The shear force is applied at the inner transverse stiffener. The girder is supported in lateral direction at the transverse stiffeners to avoid lateral torsional buckling.

To investigate the structural behaviour of hybrid girders and the plastic stress redistribution possibilities, the material models applied has special importance. Therefore, the most accurate and up-to-date material models are selected for this study, which employs a linear elastic – hardening plastic material model with von Mises yield criterion. To model NSS, a quad-linear material model following the guidelines in prEN 1993-1-14 is applied. This material model has been proposed at first by Yun et al. (2019) and it has been developed based on statistical evaluation of many coupon test results. The material model is written in form of Eqs. (9)-(14).

$$\sigma(\varepsilon) = \begin{cases} E\varepsilon, & \varepsilon \leq \varepsilon_{\mathrm{y}} \\ f_{\mathrm{y}}, & \varepsilon_{\mathrm{y}} < \varepsilon \leq \varepsilon_{\mathrm{sh}} \\ f_y + E_{\mathrm{sh}}(\varepsilon - \varepsilon_{\mathrm{sh}}), & \varepsilon_{\mathrm{sh}} < \varepsilon \leq C_1\,\varepsilon_{\mathrm{u}} \\ f_{\mathrm{y}}C_1\varepsilon_{\mathrm{u}} + \dfrac{f_{\mathrm{u}} - f_{\mathrm{y}}C_1\varepsilon_{\mathrm{u}}}{(\varepsilon_{\mathrm{u}} - C_1\varepsilon_{\mathrm{u}})}(\varepsilon - C_1\varepsilon_{\mathrm{u}}), & C_1\,\varepsilon_{\mathrm{u}} < \varepsilon \leq \varepsilon_{\mathrm{u}} \end{cases} \tag{9}$$

$$E_{\mathrm{sh}} = \frac{f_{\mathrm{u}} - f_{\mathrm{y}}}{C_2\varepsilon_{\mathrm{u}} - \varepsilon_{\mathrm{sh}}} \tag{10}$$

$$\varepsilon_{\mathrm{sh}} = 0.1\frac{f_{\mathrm{y}}}{f_{\mathrm{u}}} - 0.055\ but\ 0.015 \leq \varepsilon_{\mathrm{sh}} \leq 0.03 \tag{11}$$

$$\varepsilon_{\mathrm{u}} = 0.6\left(1 - \frac{f_{\mathrm{y}}}{f_{\mathrm{u}}}\right), but\ 0.06 \leq \varepsilon_{\mathrm{u}} < A \tag{12}$$

$$C_1 = \frac{\varepsilon_{\mathrm{sh}} + 0.25(\varepsilon_{\mathrm{u}} - \varepsilon_{\mathrm{sh}})}{\varepsilon_u} \tag{13}$$

$$C_2 = \frac{\varepsilon_{\mathrm{sh}} + 0.4(\varepsilon_{\mathrm{u}} - \varepsilon_{\mathrm{sh}})}{\varepsilon_{\mathrm{u}}} \tag{14}$$

To model HSS structural parts, Ramberg-Osgood-type material model is applied defined by Eqs. (15)-(16).

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{f_y}\right)^n \tag{15}$$

(16)

where: f_y is $y$ield strength [N/mm^2], E is the Young´s modulus of steel [N/mm^2], ε is the strain [mm/mm], σ is the stress [N/mm^2], *n=14* is a coefficient taken by the recommendation of prEN 1993-1-14.

Imperfections have also an important role in the numerical simulations since the structural behaviour and the failure mode is governed by the buckling of the web in the elastic or plastic branch. Therefore, special attention is given to the applied imperfection shape and magnitude. Initial imperfections are the geometrical and structural imperfections (residual stresses) which can be modeled by equivalent geometric imperfections. Within the current study, both versions were studied, but for keeping simplicity, the application of the equivalent geometric imperfections is presented in the current paper. There are different alternatives to define the equivalent geometric imperfections: (i) first eigenmode shape magnified and (ii) hand-defined imperfections by changing the original geometry of the perfect model. The latter is applied in the current study. Imperfections on both the web and flanges are applied. The imperfection magnitude equal to h_w/200 is applied on the web plate and b_f/2/50 on the flanges according to the rules of prEN1993-1-14. Both imperfections are applied as sinusoidal shape along the girder length as well as along the web depth.

Mesh sensitivity study and model validation are also preformed on the numerical model to prove its accuracy. The aim of the mesh sensitivity study is to prove, that the applied numerical model uses an FE mesh size which is dense enough to accurately capture the exact mathematical solution. The aim of the model validation is to check, if the numerical model is capable of accurately capture the physical phenomenon (failure mode) analyzed. The details of the numerical model verification and validation can be found in previous papers of the authors separately for NSS and HSS structures. The same modelling methodology is applied also in the current analysis and used to model hybrid girders.

3.2 Investigated parameter range

In the numerical analysis the following parameter domains are analyzed, which has a significant importance from the point of view of the validity interval of the studied resistance models. In total, more than 200 different cross-section geometries are investigated, which contained all cross-section classes (classes 1-4). The studied cross-sections were selected from the following parameter ranges:

- h_w 400 – 1200 mm (15.7 – 47.2 inches),
- t_w 2 – 15 mm (0.08 - 0.6 inches),
- b_f 200 – 450 mm (7.9 – 17.7 inches),
- t_f 12 – 30 mm (0.47 – 1.18 inches),
- h_w/t_w 50 – 200,
- b_f/t_f 7.5 – 22.5,
- $f_{y,f}$, $f_{y,w}$ 235 MPa – 700 MPa (34 – 101 ksi),
- $f_{y,f}$ / $f_{y,w}$ 1 - 3
- $\alpha=a/h_w$ 1.2-1.5,

where $f_{y,f}$ / $f_{y,w}$ is the ratio of the flange and web material's yield strength, α is the aspect ratio of the web panel. All the other notations are shown in Fig. 1. The cross-sections on which the bending resistances are calculated are selected to cover class 1-4 cross sections. The cross-sections where the M-V interactions are analyzed are so selected, that the web fulfills the sensitivity requirement of the EN 1993-1-5 against shear buckling defined by Eq. (17).

$$\frac{h_w}{t_w} \leq \frac{72 \cdot \varepsilon}{\eta} = 48.82, \quad (17)$$

4. Bending moment resistance

At first the bending pure moment resistance is investigated. For one specific cross-section with web of 1000x25 mm, flanges of 260x25 mm, panel length of 1500 mm, the moment – displacement curves are presented in Fig. 5. The analyzed cross-section belongs to class 1 according to EN 1993-1-1, so the plastic bending resistance can be utilized. The left diagram shows, the bending moment - displacement curves for 5 different girders using different steel grades for the flanges. The steel grade of the web is always kept constant. The right graph shows the same curves just divided by the analytically calculated bending moment resistances.

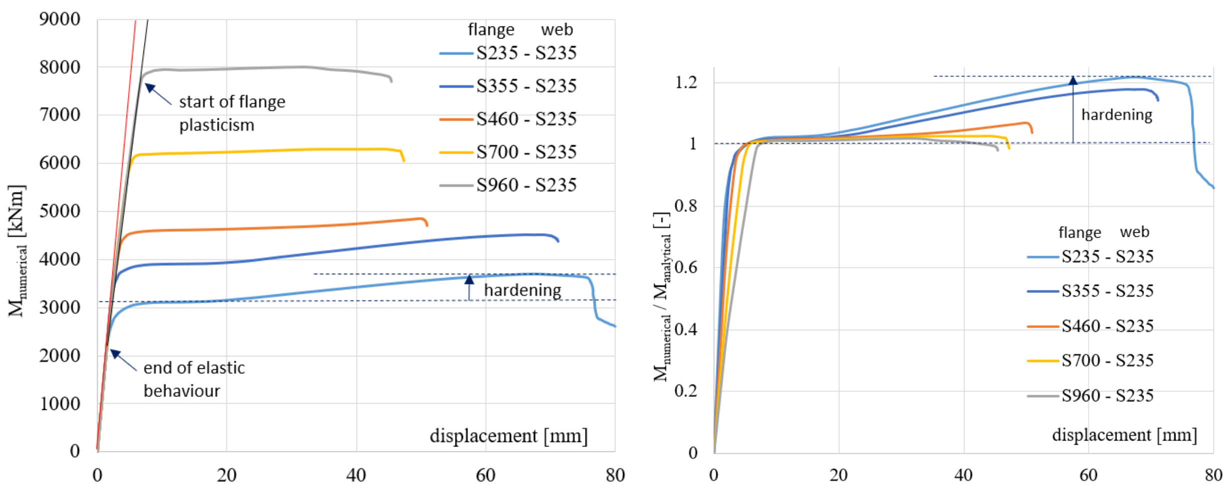


Figure 5: Moment – displacement curves for class 1 specimens allowing plastic design.

Results show, by increasing the flange steel grade, the bending moment resistance significantly increases and the ductility decreases. In the case of hybrid girders the moment – deformation diagram has a knick-point by reaching the yield strength of the NSS web (indicated by "end of elastic behaviour" in Fig. 5a). After this point, the load-deformation graph will have a reduced stiffness following a quasi-linear relationship till reaching the yield strength of the flanges, which happens at different bending moment levels for the different analyzed specimens. This phenomenon is presented by the difference of the black and red lines in Fig. 5. After reaching the flange yield strength, NSS cross-sections have significant hardening rate, which reduces by increasing the steel grade of the flanges. These results are completely aligned with the previous results of Zhu et al. (2023) proposing to introduce stricter criteria for class 1 cross-section allowing plastic design. Similar results are presented in Fig. 6 for class 3 cross-sections having a web of 1000x10 mm, flanges of 250x13-22 mm, panel length of 1500 mm. In this case, the flange thickness is changed to ensure the same $b_f/f_t/\varepsilon_f$ ratio for all specimens (where: ε_f is $\sqrt{(235\ \text{MPa} / f_y)}$). Numerical results proved, the ductility of the specimens significantly decreases, if cross-section geometry is kept the same and the flange yield strength increases. However, if the $b_f/f_t/\varepsilon_f$ ratio of the cross-section is kept constant, no ductility decrease can be observed. This observation proves, the Eurocode-based approach to have limitations characterized by the $b_f/f_t/\varepsilon$ ratio is correct and could be also applied for hybrid cross-sections. The numerical results also prove, that the numerically calculated bending resistance is larger for the NSS as well as for the hybrid girders then the analytically calculated moment resistance. In the case of Fig. 5, the moment resistance is compared to the plastic moment resistance, in the case of Fig. 6, the elastic moment resistance is used by considering partially plastic behaviour within the web as shown in Fig. 2b.

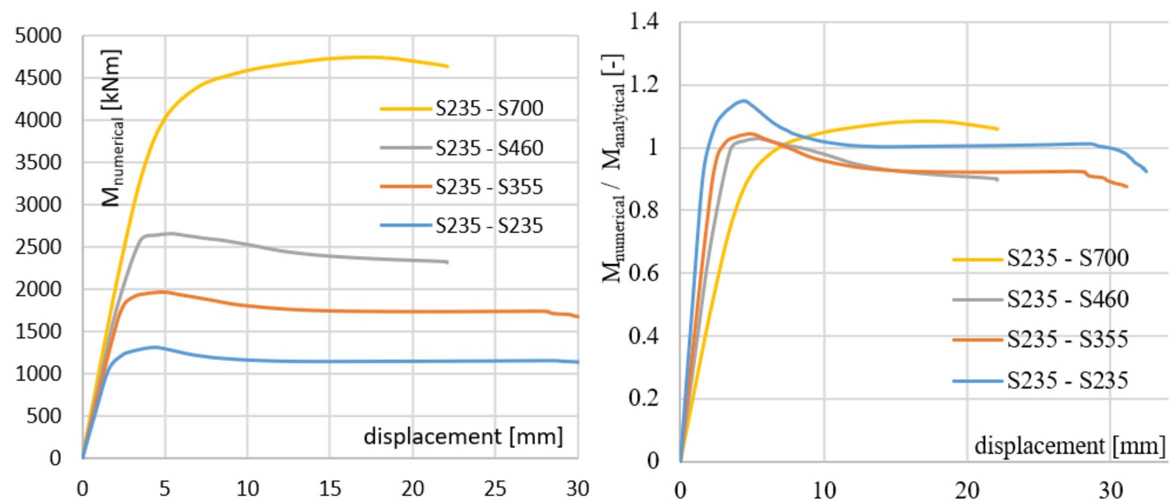


Figure 6: Moment – displacement curves for class 3 specimens allowing elastic design.

The analytically and numerically calculated bending moment resistances are also evaluated in terms of the steel grade and the web depth-to-thickness ratio. The comparison is shown in Fig. 7. The horizontal axis shows the $h_w/t_w/\varepsilon_w$ ratio for all specimens, which makes the comparison of flanges having different steel grades comparable. The vertical axis shows the ratio of the numerically and analytically calculated bending moment resistance. The left graph introduces the results of non-hybrid specimens having the same steel grade for the web and flanges. The right graph shows the results of the hybrid girders. In their notation the first number always refers to the flange yield strength and the second number to the web yield strength. The results show for all cross-section classes, the analytically calculated moment resistances are close to the numerically calculated ones even for non-hybrid and hybrid girders. No significant difference can be observed depending on the yield strength on neither diagram. The change of the elastic and plastic design is more dominant in case of non-hybrid girders than for hybrid girders. The reason of it is that in the case of hybrid girders, the elastic resistance also considers the web partial plastification, which vanishes the strict border between the elastic and plastic design. Results also prove, that the partial plastification of the web could be considered in the design of hybrid girder within the analyzed parameter range. During the study it has been always checked, that the maximum plastic strain within the web in the ultimate load level does not reach the allowed plastic strain (5%). Similar comparison is made for a large database; results are shown in Fig. 8. This graph shows all the numerically calculated bending resistances compared to the analytical ones for all class 1-2-3-4 cross-sections. It can be seen, that the above introduced analytical resistance models gives highly accurate resistances for the hybrid girders and they could be used safety in the design. The highest difference and small resistance underestimation (max. 3-5%) are observed in the case of class 4 cross-sections having relatively large web slenderness ratio. Therefore, further investigation of this parameter domain is suggested as future research task.

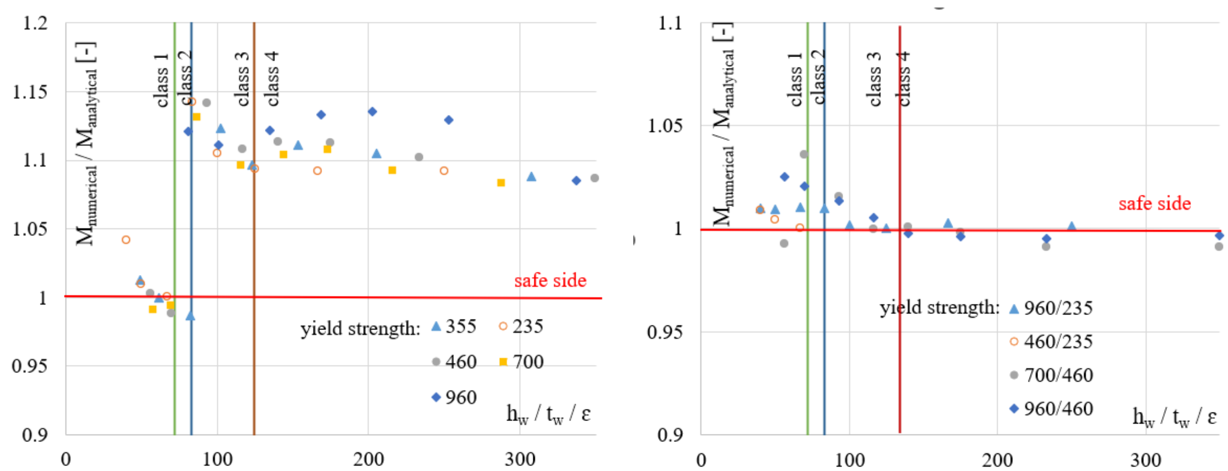


Figure 7: Comparison of numerical and analytical bending resistances depending on the web plate slenderness and steel grade: a) non-hybrid girders, b) hybrid girders.

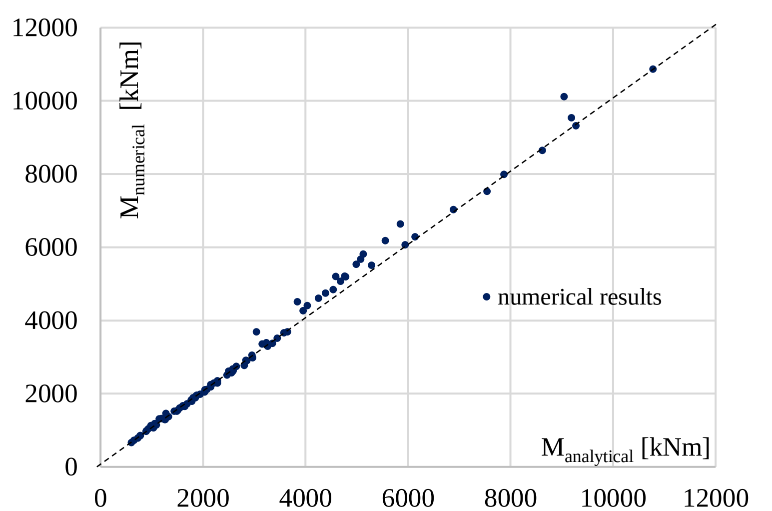


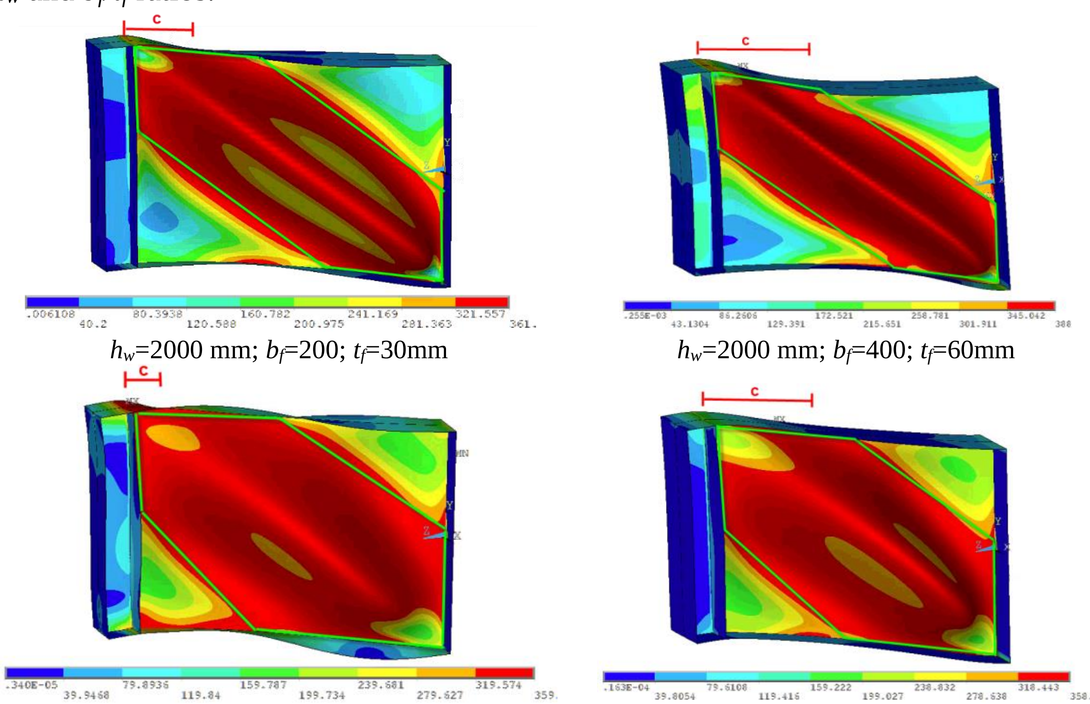
Figure 8: Comparison of numerical and analytical bending resistances.

5. Shear buckling resistance

Within the numerical parametric study to investigate the shear buckling resistance of hybrid girders, the web material is always kept constant (S235 – f_y=235 MPa) and the flange material is increased up to S700 (f_y=700 MPa). The shear buckling resistance contains the contribution of the web ($V_{bw,Rd}$) and the flanges ($V_{bf,Rd}$) as given by Eqs. (1)-(3). Within the numerical model, the separation of these two resistances is obviously not possible, but results can be evaluated depending on the flange contribution, which is made by the authors. By using Eq. (18) and the numerically calculated resistances ($V_{numerical}$), the shear buckling reduction factor is back-calculated and compared to the shear buckling curve of EN 1993-1-5.

$$V_{numerical} = \frac{\chi_w \cdot f_y \cdot h_w \cdot t_w}{\sqrt{3} \cdot \gamma_{M1}} + \frac{b_f \cdot t_f^2 \cdot f_{yf}}{c \cdot \gamma_{M1}} \left(1 - \left(\frac{M}{M_{f,Rd}}\right)^2\right) \tag{18}$$

In the recent years, researchers proved the flange contribution resistance model needs improvement, and its modification can be efficiently made through the parameter c according to Eq. (4) because it considers both the flange and web yield strengths. Within the Eurocode-based design model, the flange contribution is calculated based on a mechanism, where four plastic hinges are formulated in the flanges as shown in Fig. 3. By using the presented mechanical model, the shear resistance is calculated by dividing the plastic moment resistance of the flanges by the distance of the plastic hinges (c). To illustrate the physical meaning of this parameter, typical failure modes and von-Mises stress distributions are presented in Fig. 9 demonstrating the plastic hinge development and the value of *c* for different girder geometries having significantly different h_w/t_w and b_f/t_f ratios.



h_w=2000 mm; b_f=200; t_f=30mm | h_w=2000 mm; b_f=400; t_f=60mm

h_w=1000 mm; b_f=200; t_f=20mm | h_w=1000 mm; b_f=400; t_f=40mm

Figure 9: Typical failure modes and stress distributions as shown by Jáger et al. (2019).

The value of the parameter *c* considers the yield strength ratio of the flange and the web plate, therefore, it could handle the specialties of hybrid girders. Therefore, within the current study the accuracy of the different c parameter calculation methods is investigated, compared and their differences are highlighted. The back-calculation of the shear buckling reduction factor (χ_w) is made therefore, by using four different c parameter calculation methods, as given by Eqs. (19)-(22). The results are shown in Fig. 10, accordingly. The horizontal axis of the graph shows the relative slenderness ratio regarding shear buckling. The vertical axis shows the back-calculated shear buckling reduction factor. The shear buckling curve of the EN 1993-1-5 considering rigid end-post is also presented by black lines in all the four graphs. The datapoints present the back-calculated results from the numerical analysis.

proposal of EN 1993-1-5: $$c = a \cdot \left(0.25 + \frac{1.6 \cdot b_f \cdot t_f^2 \cdot f_{yf}}{t_w \cdot h_w^2 \cdot f_{yw}}\right) \tag{19}$$

proposal of Jáger et al. (2019): $$c = a \cdot \left(0.1 + \frac{8.5 \cdot b_f \cdot t_f^2 \cdot f_{yf}}{t_w \cdot h_w^2 \cdot f_{yw}}\right) \tag{20}$$

proposal of Pedro et al. (2022): $$c = a \cdot 1.6 \cdot \left(\frac{b_f \cdot t_f^2 \cdot f_{yf}}{t_w \cdot h_w^2 \cdot f_{yw}}\right)^{0.44} \tag{21}$$

current proposal: $$c = a \cdot 1.6 \cdot \left(\frac{b_f \cdot t_f^2}{t_w \cdot h_w^2}\right)^{0.44} \frac{f_{yf}}{f_{yw}} \tag{22}$$

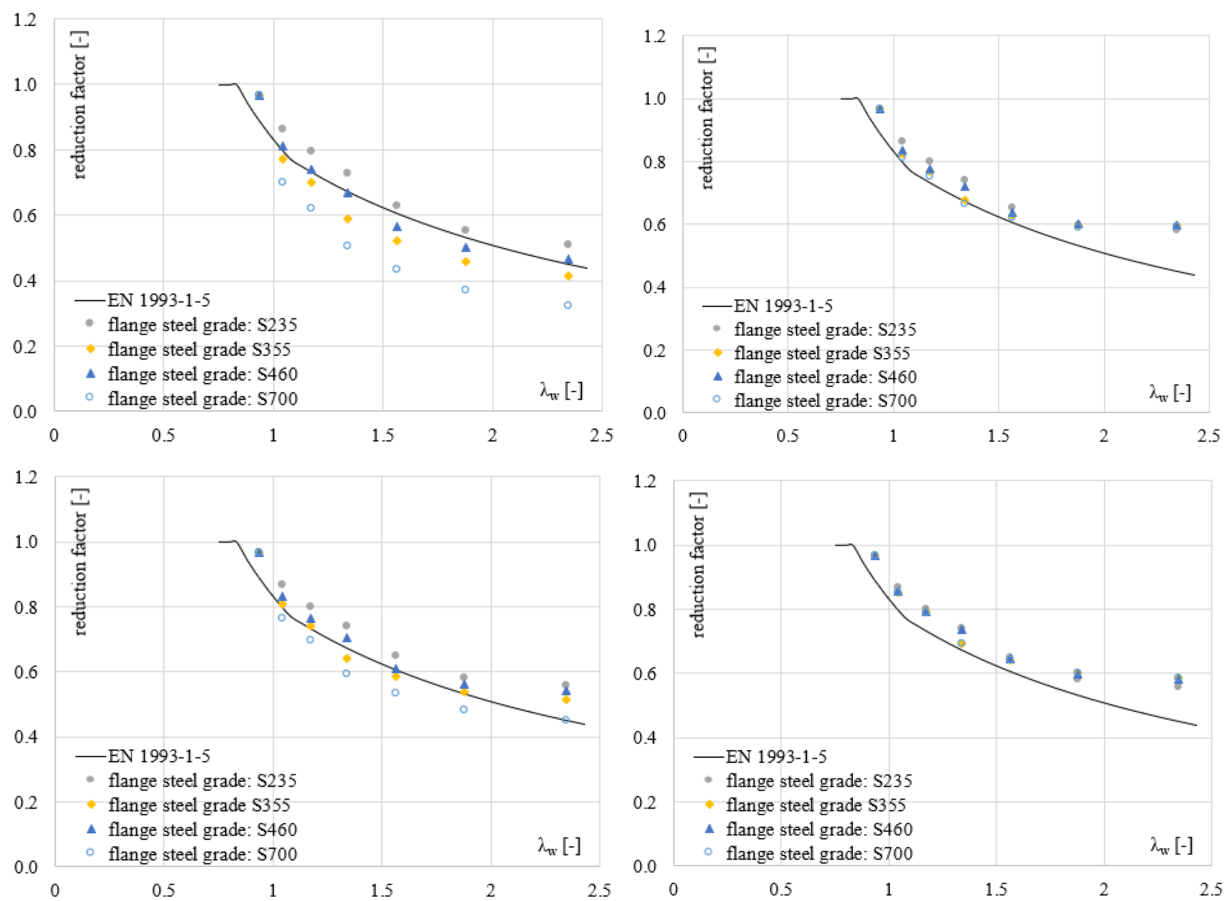


Figure 10: Back-calculated shear buckling reduction factors depending on the web slenderness ratio and the yield strength, evaluation based on a) current EN 1993-1-5, b) proposal of Jáger et el. c) proposal of Pedro et al., d) current proposal.

Results prove the current EN 1993-1-5 proposal does not provide accurate results if the flange yield strength is increased; the calculated results would be significantly on the unsafe side for hybrid girders (data points located belove the shear buckling curve on the unsafe side). The latest proposal of Pedro et al. (2022), which is proved currently to be the most accurate proposal for non-hybrid girders, could be also not used for hybrid girders. However, it is observed by the authors, the parameter c should linearly depend on the ratio of the flange and web yield strengths. Therefore, Eq. (21) is enhanced to Eq, (22). Results presented in Fig. 10 shows, there is only a negligible difference in the back-calculated values for the non-hybrid and hybrid girders. It proves the accuracy of the proposed design equation. Similar results for the entire database investigated can be found in Fig. 11 regarding the back-calculated shear buckling reduction factor. This comparison proves, all results would be on the safe side by using the modified design equation. The comparison of the numerically and analytically calculated shear buckling resistances is also shown in Fig. 12. Results prove, the improved design model provides safe sided shear buckling resistances, which match the numerical results with large accuracy.

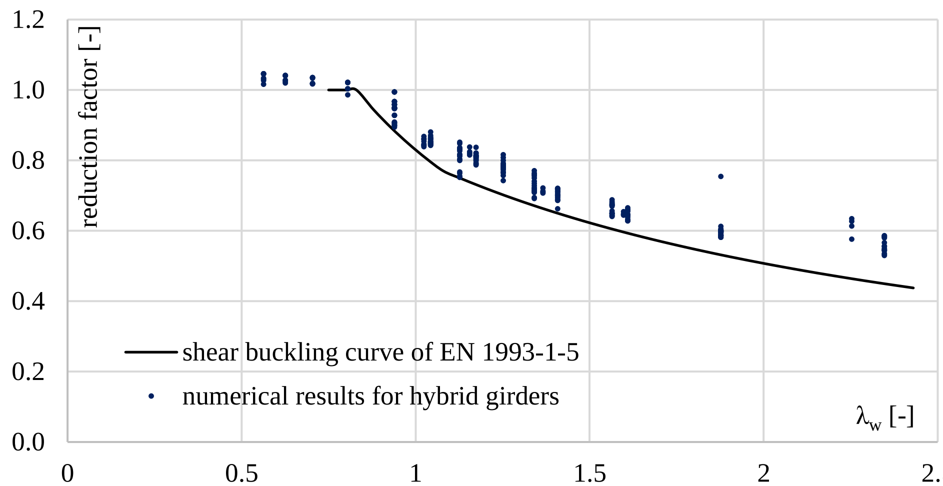


Figure 11: Comparison of the numerically back-calculated and analytical shear buckling reduction factor.

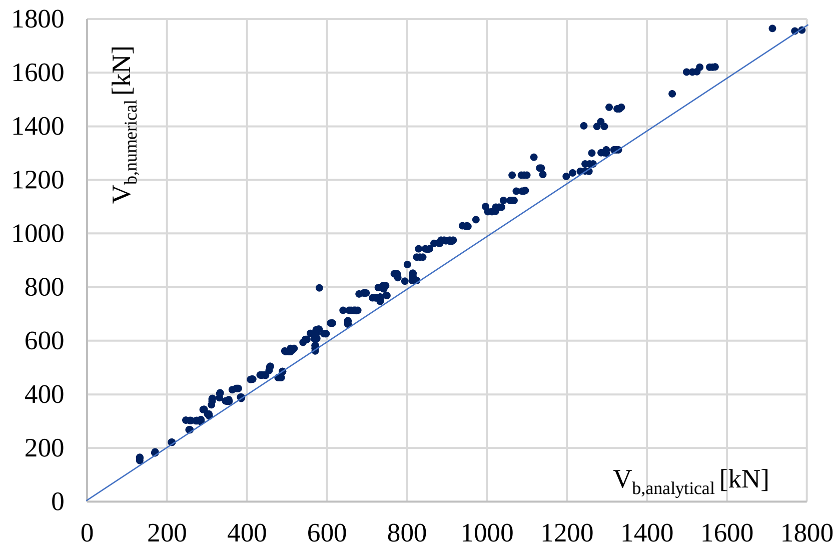


Figure 12: Comparison of numerical and analytical shear buckling resistances.

6. Interaction of bending and shear buckling resistance

Finally, the M-V interaction behaviour of the studied I-girders is evaluated. Results for a non-hybrid and a hybrid girder are presented in Fig. 13, which clearly show the differences within the interaction behaviour of non-hybrid and hybrid girders. The horizontal axis of the graph shows the applied bending moment, the vertical axis shows the shear force. Four resistance values are also indicated on the graphs: (i) shear buckling resistance of the web alone ($V_{w,Rd}$), (ii) shear buckling resistance of the entire cross-section – web plus flange contributions ($V_{w,Rd}$+ $V_{f,Rd}$) considering the new proposed *c* value, (iii) bending moment resistance of the flanges alone ($M_{f,R}$), (iv) elastic bending moment resistance of the full cross-section ($M_{el,R}$). The black continuous line represents the M-V interaction diagram given by Eqs. (1)-(3) and Eqs. (7)-(8). The datapoints marked by blue represents the numerical results; if the numerically calculated resistances are lying outside of the interaction diagram, the design method represents safe side solutions.

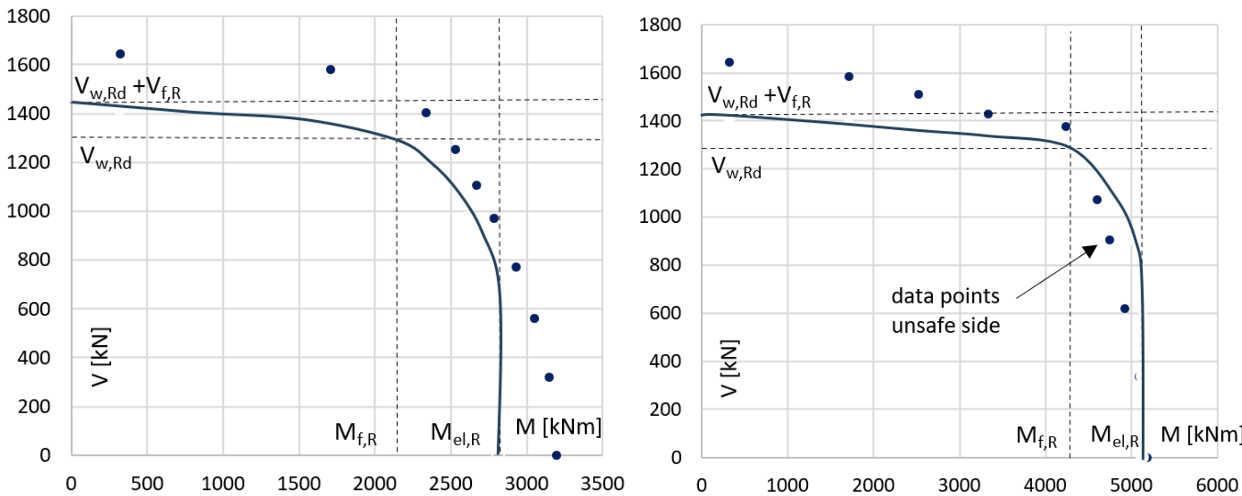


Figure 13: M-V interaction resistance: a) non-hybrid girder – S235; b) hybrid girder having web of S235 and flanges of S460 steel grades.

The left graph shows the results of a non-hybrid girder made of steel grade of S235. All data points are located outside of the Eurocode-based M-V interaction diagram. It means, the design resistance models are on the safe side and follows quite well the interaction behaviour of the girder under investigation. The right graph shows the results for the same cross-section, just increasing the flange steel grade to S460, representing hybrid rate of ~2.0. It can be seen, that the difference in the pure shear buckling resistance is almost the same compared to the non-hybrid girder, which is due to the modified c parameter as presented in Section 5 above. The pure bending moment resistance calculated by considering the partial web plastification leads also safe side moment resistance. Furthermore, numerical results proved, a more strict M-V interaction equation would be required to safely consider the structural behaviour of the hybrid girders. It means, the Eqs (7)-(8) should be also investigated and modified in the future to provide accurate and safe-sided resistance models. Similar results are observed by investigating many different cross-section geometries and hybrid rate up to 3.0. The difference between the analytical and numerical results increases by increasing the ratio of the flange and web steel material.

7. Conclusions

A systematic numerical parametric study is executed to investigate the structural behaviour of hybrid steel I-girders containing NSS web and HSS flanges. The paper presented the obtained results regarding the (i) bending moment, (ii) shear buckling and (iii) the interaction buckling resistance. Results proved the bending moment resistance of hybrid girders can be calculated by the resistance model presented in the international literature for class 1-3 cross-sections. For shear buckling, an enhanced resistance model is proposed considering the hybrid rate of the girder. Regarding M-V interaction behaviour differences between non-hybrid and hybrid girders are highlighted and specialties in the structural behaviour are presented.

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