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Methods to improve the computational efficiency of geometric nonlinear analyses of steel frames

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Abstract
A relationship was discovered between the amplification factor and the number of load increments that are needed to limit the relative error to one percent in a second-order elastic analysis with a predictor-corrector solution scheme. Previous research by the authors proposed a design equation to determine the required minimum number of load increments based on an evaluation of the elastic critical buckling load ratio. Further research has shown that an approximate amplification factor equation that is based on the B_2 multiplier equation produces similar results when the amplification factor is less than approximately four. Fifteen moment frames are used to verify the use of the new approximate amplification factor in the proposed design equation. Discussion is provided on when the new amplification factor can be used effectively to determine the required minimum number of load increments in a second-order elastic analysis.

1. Introduction
The strength requirements of frames are often evaluated considering geometric nonlinear effects, which requires the engineer to make decisions about the required modeling effort and its associated computational time to achieve a desired level of accuracy. For steel frames modeled with beam elements, these nonlinear effects are accounted for using a solution scheme that incrementally applies the loads to approximate the ‘exact’ equilibrium of the frame in the deformed configuration. The accuracy in modeling the frame in this configuration is dependent upon the number of load increments that are used to apply the external loads. Increasing the number of load increments to improve accuracy often comes at the cost of increased computational time since frame models often have a large number of degrees of freedom and multiple load combinations to consider. The effects of nonlinear material behavior may also need to be considered, but since the majority of routine building design considers only elastic material behavior (Ziemian and Ziemian 2021), this other contributing influence on the number of load increments is ignored in the present study.

The number of load increments that are necessary to achieve a 1% relative error in a second-order elastic analysis was previously evaluated by the authors (Faramawi and Rosson 2024). A total of 26 frames were modeled with an initial geometric imperfection of H/500 and increment size of 0.001 in a predictor-corrector solution scheme to obtain the ‘exact’ lateral displacement results at the top of the frame. It was found that the amplification factor (AF) of the frame can be used to determine the minimum number of load increments. Using the frame’s elastic buckling load factor

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α_{cr} to approximate the amplification factor $\mathrm{AF}_{\alpha_{cr}}$ (Merchant 1954, Eurocode EN 1993-1-1 2005, and AS 4100 2020), the displacement results using the number of increments equal to $5\mathrm{AF}_{\alpha_{cr}}\ -2$ were compared with the 'exact' displacement results and found to be within a 1% relative error.

This study explores the use of an alternative method to approximate the amplification factor using the B_2 multiplier equation in AISC 360, Appendix 8 (2022). The approximate amplification factor $\mathrm{AF}_{\mathrm{B2}}$ is calculated by performing a first-order analysis, then the interstory drift values are updated based on the appropriate B_2 multiplier at each level to approximate the second-order displacement at the top of the frame. It also investigates larger amplification factors than previously studied by increasing the gravity loads on 11 multi-story benchmark frames.

The frames were modeled in the MASTAN2 (2022) analysis software, which accounts for second-order effects using an Updated Lagrangian formulation, and for this study, the predictor-corrector solution scheme. The software is also capable of performing a linear buckling analysis using the inverse iteration method (McGuire *et al*. 2000). All members were modeled as planar 6-dof line elements with elastic material behavior and all frame models have out-of-plumb geometries.

2. Frame Amplification Factors

Numerous second-order elastic analyses were conducted to determine the minimum number of load increments that were needed to limit the relative error to 1% or less. Frames were modeled with an initial geometric imperfection of H/500 and an increment size of 0.001 in a predictor-corrector solution scheme to obtain the 'exact' results. The amplification factor was evaluated for each analysis condition using Eq. 1, where δ_{2nd} is the lateral displacement of the top left node from a second-order elastic analysis, and δ_{1st} is the displacement at the same location from a first-order analysis.

$$\mathrm{AF} = \frac{\delta_{2nd}}{\delta_{1st}} \tag{1}$$

The 'exact' amplification factor AF can be approximated by $\mathrm{AF}_{\alpha_{cr}}$ using the elastic buckling load ratio of the frame α_{cr} as given in Eurocode EN 1993-1-1 (2005) and AS 4100 (2020). The critical buckling load $\mathrm{P_{cr}}$ in Eq. 3 is obtained using closed-form equations for simple frames or from an eigenvalue analysis for more complex frames, and P is the applied load on the frame. When performing an elastic critical load analysis in MASTAN2, α_{cr} is the Mode #1 Applied Load Ratio.

$$\mathrm{AF}_{\alpha_{cr}} = \frac{1}{1-{}^{1}\!/_{\alpha_{cr}}} \tag{2}$$

$$\alpha_{cr} = \frac{\mathrm{P_{cr}}}{\mathrm{P}} \tag{3}$$

The B_2 multiplier in Eq. 4 is from Appendix 8 of AISC 360 (2022) and is used to account for the P-Δ effect of each story. In this application, the B_2 multiplier is calculated for each story, where α is taken as 1, P_{story} is the total axial load supported by the story (ΣP_{i}), and $P_{e\ story}$ is the elastic critical buckling strength of the story in the direction of translation being considered.

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\ story}}} \geq 1 \tag{4}$$

The elastic critical buckling load can be determined by an eigenvalue analysis or by using Eq. 5. In this equation, H represents the total story shear in the direction of translation (ΣV_{i}), L is the height of the story, and Δ_H is the first-order interstory drift. R_M is taken as 0.85 for moment frames. Fig. 1 illustrates the components of Eqs. 4 and 5.

$$P_{e\ story} = R_M \frac{HL}{\Delta_H} \tag{5}$$

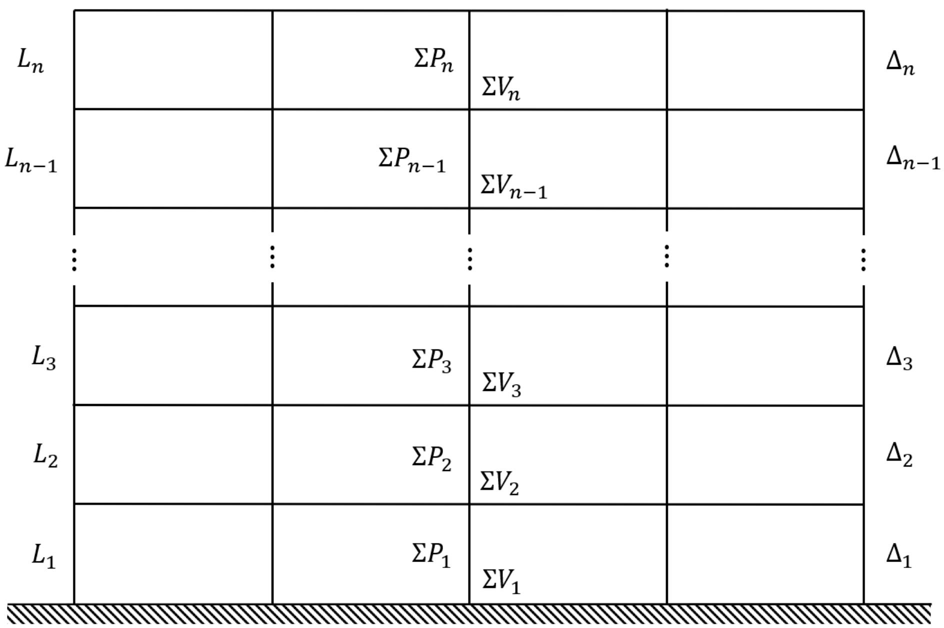


Figure 1: Frame with B_2 equation components

Using first-order analysis results of the applied loads on the frame, Eq. 6 is used to convert the lateral displacement at each level (δ_i) to the corresponding interstory drift values (Δ_i).

$$\begin{bmatrix} 1 & & & & & \\ -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \\ & & & & -1 & 1 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \vdots \\ \delta_{n-1} \\ \delta_n \end{Bmatrix} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{Bmatrix} \tag{6}$$

The $C_{i,i}$ coefficients in Eq. 7 follow from the B_2 multiplier equation and are used to convert the first-order drift values to approximate second-order interstory drift values. The variables in Eq. 7 are illustrated in Fig. 1.

$$C_{i,i} = \frac{1}{1 - \frac{\Sigma P_i \Delta_i}{0.85 \Sigma V_i L_i}} \quad (7)$$

Multiplying the first-order drift values by a diagonal matrix of the $C_{i,i}$ coefficients results in the approximate second-order interstory drift values (Δ'_i).

$$\begin{bmatrix} C_{1,1} & & & & & \\ & C_{2,2} & & & & \\ & & C_{3,3} & & & \\ & & & \ddots & & \\ & & & & C_{n-1,n-1} & \\ & & & & & C_{n,n} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{Bmatrix} = \begin{Bmatrix} \Delta'_1 \\ \Delta'_2 \\ \Delta'_3 \\ \vdots \\ \Delta'_{n-1} \\ \Delta'_n \end{Bmatrix} \quad (8)$$

The sum of the second-order drift values ($\Sigma\Delta'_i$) gives an approximation for δ_{2nd} in Eq. 1, and since δ_{1st} is the same as δ_n, the approximate amplification factor AF_{B2} is given as

$$\mathrm{AF}_{B2} = \frac{\Sigma\Delta'_i}{\delta_n} \quad (9)$$

3. Minimum Number of Increments in a Second-Order Elastic Analysis

All frames were modeled with an initial geometric imperfection of H/500 and an increment size of 0.001 in a predictor-corrector solution scheme to obtain the ‘exact’ results. Four different load magnitudes were modeled for each frame stiffness condition. Frames 1 through 3 use γ = 1, 8, 24 and correspond with frame designations A, B, and C, respectively.

$$\gamma = \frac{I_B L_C}{I_C L_B} \quad (10)$$

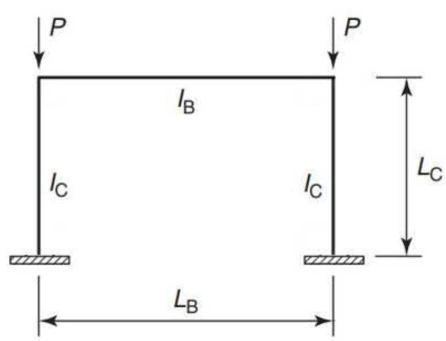


Figure 2: Frame 1 properties

As indicated in Fig. 3 with Frame 1, a linear relationship exists between the minimum number of load increments that are needed to keep the relative error below 1% and the ‘exact’ amplification factor AF. A regression analysis of the data revealed a very strong linear relationship (red line, r^2 = 0.9987). With a slope of approximately 5 and y-intercept of approximately 3, Eq. 11 was proposed for design purposes to determine the minimum number of load increments to use in a second-order elastic analysis with the predictor-corrector solution scheme (Faramawi and Rosson 2024). It conservatively uses 2 for the y-intercept because only the integer result is used, and it also ensures the relative errors remain below 1%.

$$\text{Number of Increments} = 5\text{AF} - 2 \tag{11}$$

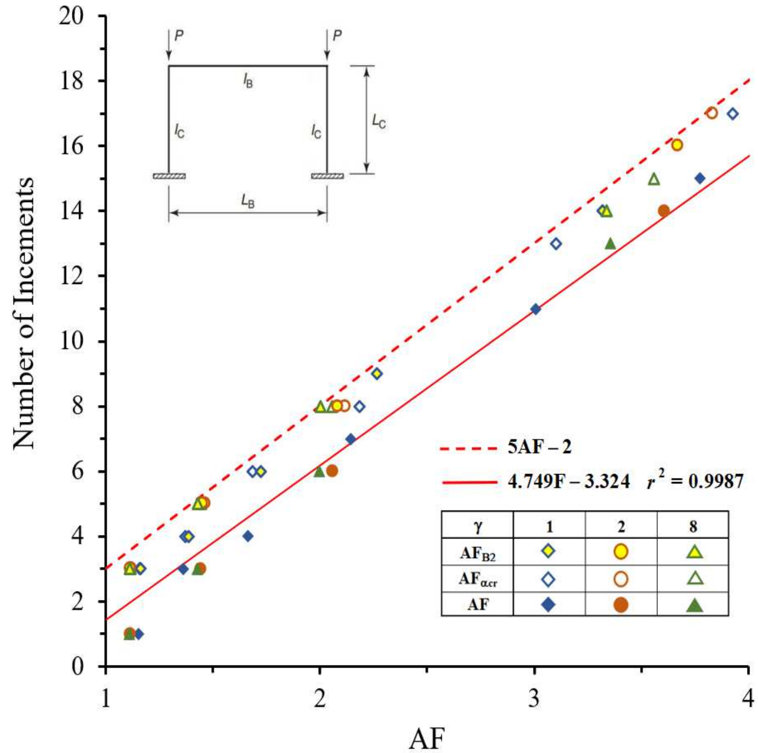


Figure 3: Number of increments vs. amplification factor for Frame 1 (relative error ≤ 1%)

As indicated by the dashed red line in Fig. 3, and the corresponding data associated with it using $AF_{\alpha_{cr}}$ and AF_{B2} in Eq. 11, the minimum number of load increments was found to produce second-order elastic results that are within 1% of the 'exact' results. Fig. 3 and data in Appendix A reveal that all the required number of increments using Eq. 11 remain above the actual minimum number of increments. Thus, Eq. 11 is found to be conservative and can safely be used for this purpose.

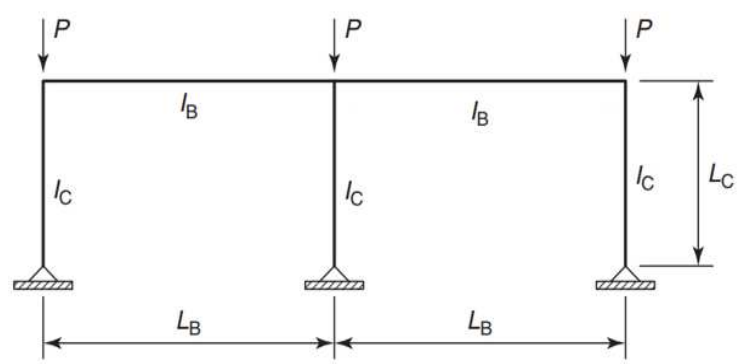


Figure 4: Frame 2 properties

As indicated in Fig. 5 with Frame 2, a similar linear relationship exists between the minimum number of load increments and the amplification factors as that given in Fig. 3 with Frame 1. A regression analysis of the data revealed a similar linear relationship (red line, $r^2 = 0.9994$) with approximately the same slope and y-intercept. As with Frame 1, the use of $\mathrm{AF}_{\alpha_{cr}}$ and $\mathrm{AF}_{\mathrm{B2}}$ in Eq. 11 was found to produce conservative results and can safely provide second-order elastic results that are within 1% of the 'exact' results.

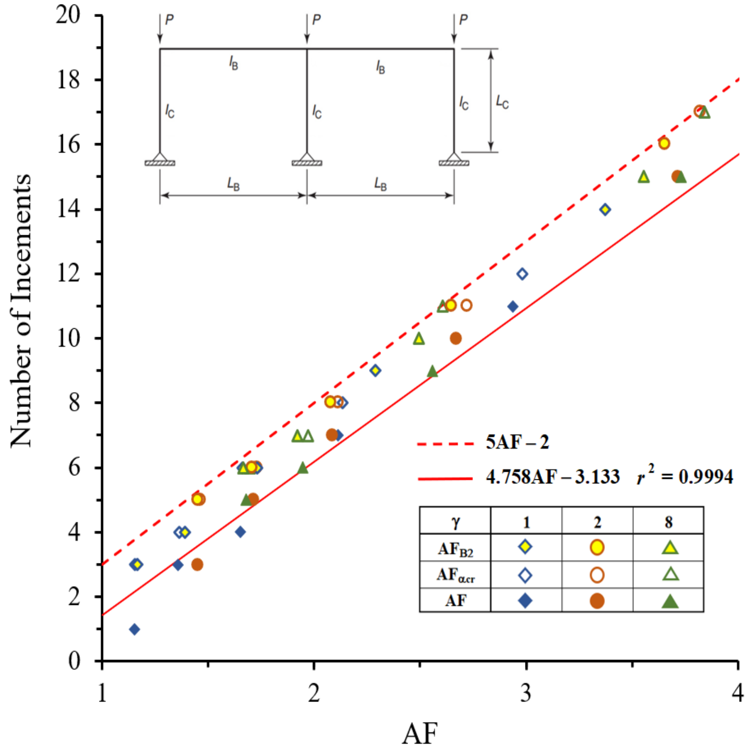


Figure 5: Number of increments vs. amplification factor for Frame 2 (relative error ≤ 1%)

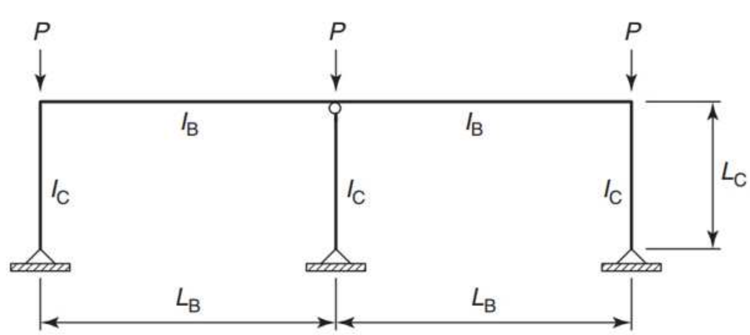


Figure 6: Frame 3 properties

Frame 3 in Fig. 6 is very similar to Frame 2, the only difference is the internal hinge at the top of the middle column. This frame was used to determine if the hinge had any effect on the results. A similar linear relationship exists between the minimum number of load increments and the amplification factor. The results in Fig. 7, and the corresponding data in Appendix B, reveal that the internal hinge has no effect on the use of Eq. 11, and it can safely be used to determine the minimum number of load increments.

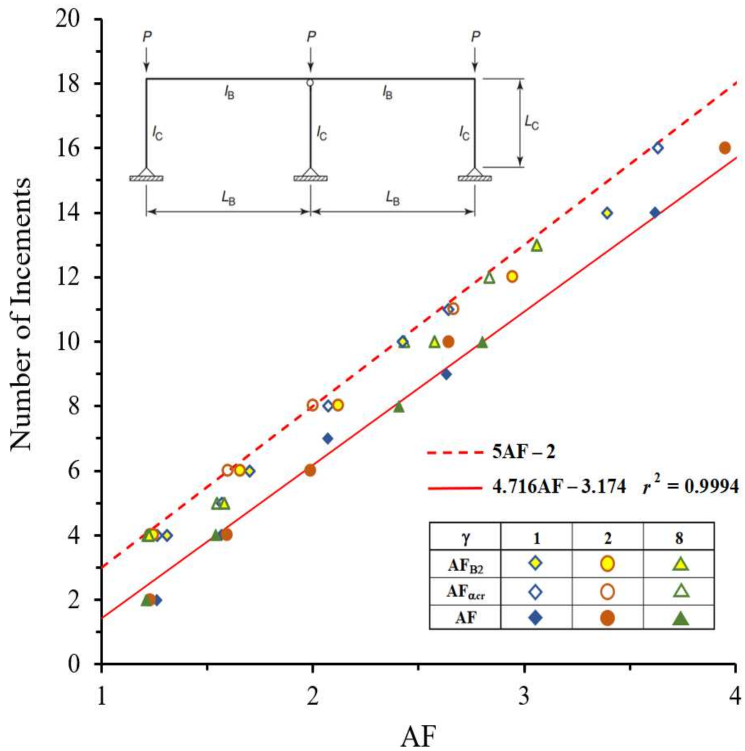


Figure 7: Number of increments vs. amplification factor for Frame 3 (relative error ≤ 1%)

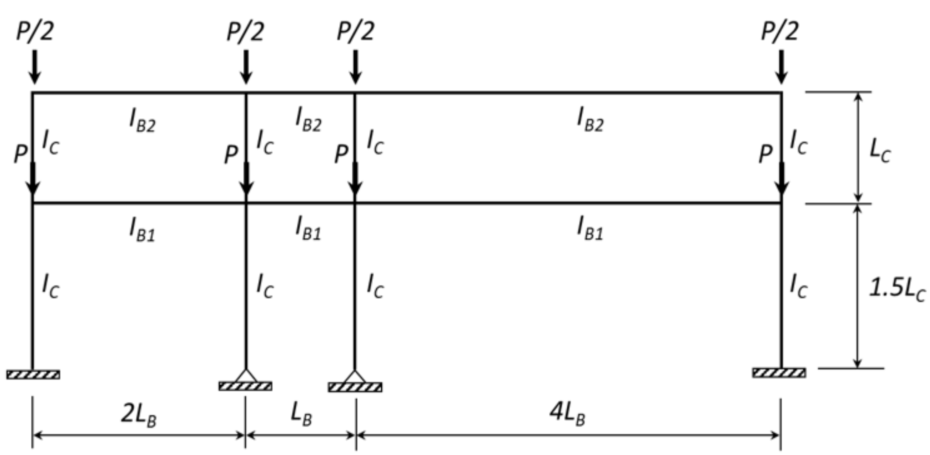


Figure 8: Frame 4 properties

Frame 4 in Fig. 8 was developed to evaluate the effectiveness of Eq. 11 on a more complex unbraced frame. A linear buckling analysis was conducted using MASTAN2 on six different beam and column stiffness configurations as indicated in Table 1. Six configurations (A through F) were used to conduct second-order elastic analyses with four magnitudes of external load for each configuration.

$$\gamma_1 = \frac{I_{B1}L_C}{I_C L_B} \tag{12}$$

$$\gamma_2 = \frac{I_{B2}L_C}{I_C L_B} \tag{13}$$

Table 1: Analysis conditions and results for Frame 4

Configuration	γ_1	γ_2	P_{cr}
A	1	2	1.352
B	8	2	1.677
C	24	2	1.768
D	0.5	1	2.406
E	4	1	3.160
F	12	1	3.426

As indicated in Fig. 9, a similar linear relationship exists between the minimum number of load increments and the amplification factor as that given in Figs. 3, 5, and 7 for Frames 1, 2, and 3, respectively. A regression analysis of the data revealed a similar linear relationship (red line, r^2 = 0.9961) with approximately the same slope and *y*-intercept. As with the previous frames, the use of $\mathrm{AF}_{\alpha_{cr}}$ and $\mathrm{AF}_{\mathrm{B2}}$ in Eq. 11 for Frame 4 was found to produce second-order elastic results that were within 1% of the 'exact' results.

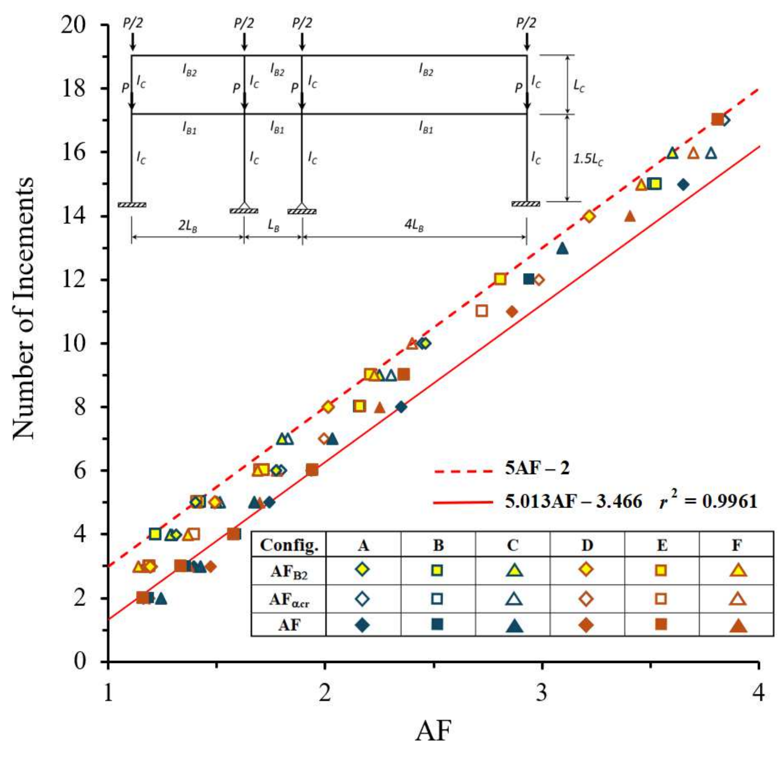


Figure 9: Number of increments vs. amplification factor for Frame 4 (relative error ≤ 1%)

4. Validation Study with 11 Benchmark Frames

With the successful utilization of Eq. 11 in the previous section, 11 moment frames that were developed by Lu *et al*. 1977, Vogel 1985, and Statler *et al*. 2011 were used to test the validity of this expression to determine the minimum number of load increments in a second-order elastic analysis using the predictor-corrector solution scheme. An overview description of Frames 5 through 15 is given in Fig. 10, and the analysis results for these frames are given in Appendix C. Frame designation A indicates the original stiffness and loads as those given in the Benchmark Problems file of MASTAN2 (2022). Frame designation B indicates the same stiffness conditions but with increased gravity loads to produce larger amplification factors for all 11 frames. This was necessary because the original loads resulted in AF values between 1.153 and 1.686, but with the increased loads the AF values ranged between 1.755 and 3.373.

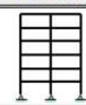
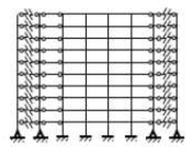
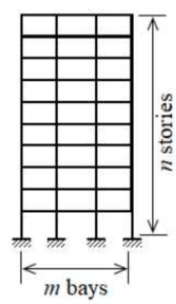
Frame No.	Description	Geometry	Load Combination
5	6 stories, 2 bays [Vogel 1985]		$1.2D + 1.0L + 0.5L_r + 1.0W$
6−9	10 stories, 3 bays; 0, 4, 8, 12 lean-ons [Lu et al. 1977; Statler et al. 2011]		$1.2D + 1.0L + 0.5L_r + 1.0W$
10	10 stories, 5 bays [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$
11	20 stories, 1 bay [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$
12	26 stories, 3 bays [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$
13	30 stories, 2 bays [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$
14	30 stories, 2 bays [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$
15	40 stories, 2 bays [Lu et al. 1977]		$1.2D + 1.0L + 0.5L_r + 1.0W$

Figure 10. Overview of benchmark Frames 5 − 15

As indicated in Fig. 11, a similar linear relationship exists between the minimum number of load increments and the amplification factor (red line, $r^2 = 0.9914$); however, there is a steeper slope to the line than before. Nonetheless, Eq. 11 was still found to produce results for all 11 frames that were within 1% of the 'exact' results. Fig. 12 illustrates the reason why this remains the case. The $\mathrm{AF}_{\alpha_{cr}}$ and $\mathrm{AF}_{\mathrm{B2}}$ values are always slightly larger than the actual AF values. This results in higher values for the minimum number of increments in Eq. 11 compared with the ‘exact’ results; thus they are conservative and always within 1% of the required minimum number of load increments. Fig. 12 also illustrates that using AF_{B2} to determine the number of increments results in slightly higher values than those produced using $AF_{\alpha_{cr}}$, especially for amplification values greater than approximately 2.

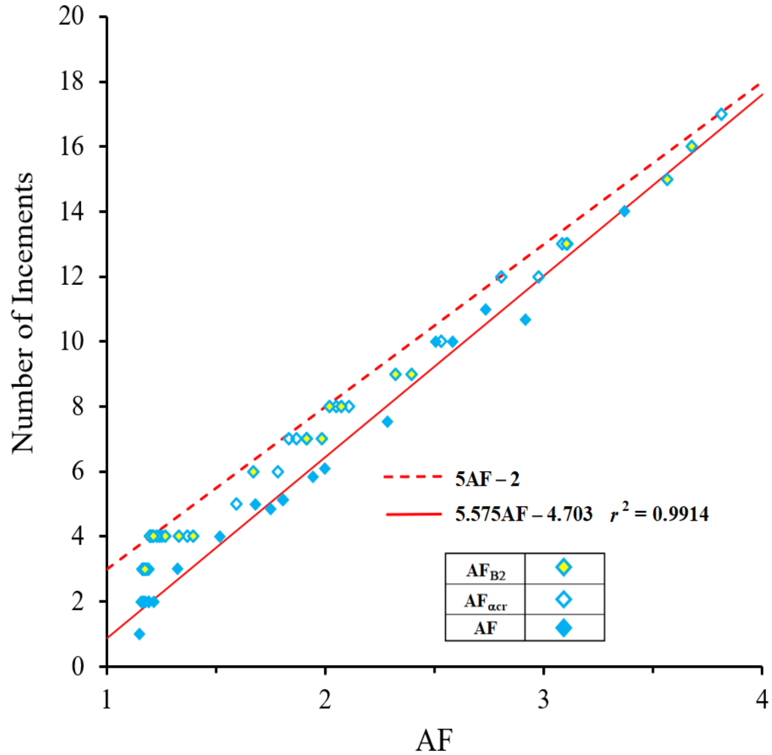


Figure 11: Number of increments vs. amplification factor for Frames 5 − 15 (relative error ≤ 1%)

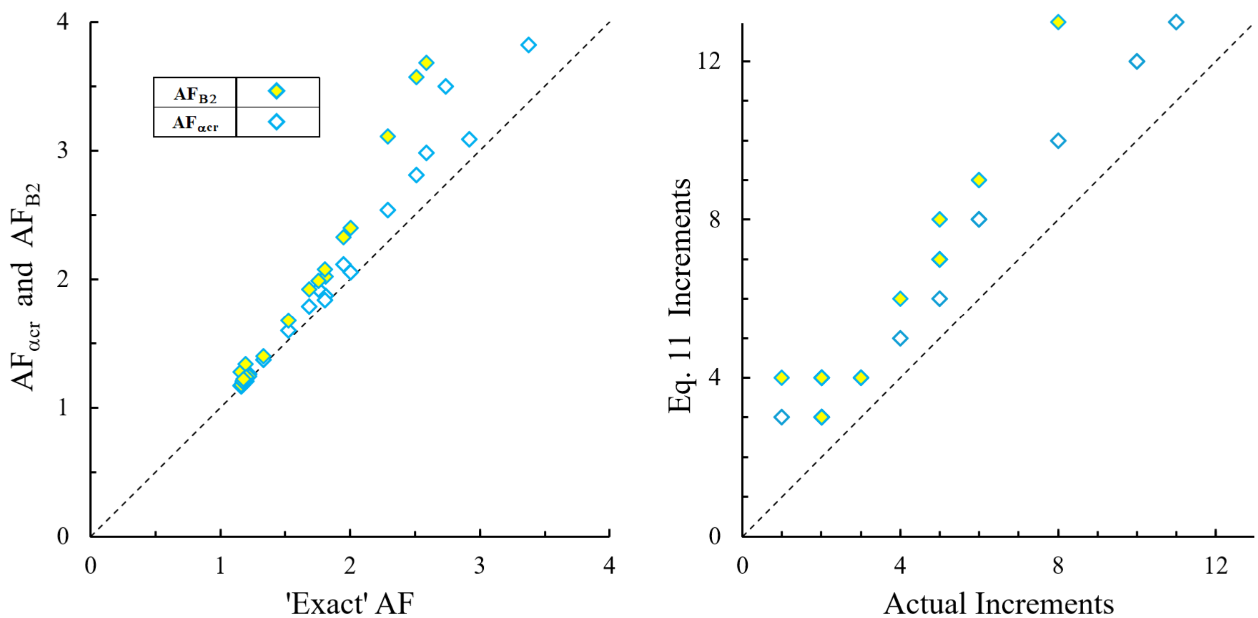


Figure 12: Comparison of amplification factors and number of increments with ‘exact’ results (Frames 5 − 15)

5. Conclusions

Based on the displacement results of the four unbraced frames and 11 benchmark moment frames, a linear relationship was discovered between the amplification factor and the number of load increments that are needed to limit the relative error to 1% in a second-order elastic analysis with a predictor-corrector solution scheme. The integer result of 5AF – 2 is proposed for routine design purposes to determine the minimum number of load increments. Since a linear buckling analysis is required to calculate $\mathrm{AF}_{\alpha_{cr}}$, an amplification factor based on the B_2 multiplier equation was investigated as an alternative. The use of $\mathrm{AF}_{\mathrm{B2}}$ was found to produce reliable and accurate results up to an amplification factor of approximately 4. The $\mathrm{AF}_{\mathrm{B2}}$ results were very comparable to those using $\mathrm{AF}_{\alpha_{cr}}$, especially for amplification factors below approximately 2. It is recommended to limit the use of $\mathrm{AF}_{\mathrm{B2}}$ to amplification factors below 4; however, since most design conditions have amplification factors well below this threshold, using $\mathrm{AF}_{\mathrm{B2}}$ to determine the minimum number of load increments in a second-order elastic analysis can be confidently and widely used in practice.

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Appendix A (Results for Frames 1 and 2)

Frame	'Exact' AF	Actual Increments	$AF_{\alpha cr}$	Eq. 11 Increments	AF_{B2}	Eq. 11 Increments
1A	1.362	3	1.372	4	1.388	4
	1.664	4	1.685	6	1.722	6
	2.142	7	2.184	8	2.267	9
	3.007	11	3.101	13	3.320	14
1B	1.114	1	1.118	3	1.116	3
	1.444	3	1.464	5	1.453	5
	2.057	6	2.118	8	2.083	8
	3.608	14	3.834	17	3.672	16
1C	1.110	1	1.114	3	1.111	3
	1.425	3	1.445	5	1.429	5
	1.997	6	2.056	8	2.001	8
	3.353	13	3.560	15	3.340	14

Frame	'Exact' AF	Actual Increments	$AF_{\alpha cr}$	Eq. 11 Increments	AF_{B2}	Eq. 11 Increments
2A	1.357	3	1.362	4	1.392	4
	1.653	4	1.663	6	1.731	6
	2.114	7	2.135	8	2.288	9
	2.939	11	2.981	12	3.373	14
2B	1.712	5	1.730	6	1.710	6
	2.085	7	2.116	8	2.078	8
	2.671	10	2.724	11	2.648	11
	3.717	15	3.822	17	3.656	16
2C	1.679	5	1.697	6	1.665	6
	1.946	6	1.973	7	1.920	7
	2.556	9	2.606	11	2.493	10
	3.730	15	3.840	17	3.556	15

Appendix B (Results for Frames 3 and 4)

Frame	'Exact' AF	Actual Increments	$AF_{\alpha cr}$	Eq. 11 Increments	AF_{B2}	Eq. 11 Increments
3A	1.566	4	1.568	5	1.699	6
	2.069	7	2.074	8	2.426	10
	2.632	9	2.641	11	3.393	14
	3.618	14	3.634	16	5.648	26
3B	1.228	2	1.231	4	1.247	4
	1.593	4	1.601	6	1.657	6
	1.988	6	2.001	8	2.122	8
	2.645	10	2.670	11	2.946	12
3C	1.212	2	1.214	4	1.225	4
	1.539	4	1.546	5	1.580	5
	2.407	8	2.430	10	2.576	10
	2.803	10	2.836	12	3.058	13

Frame	'Exact' AF	Actual Increments	$AF_{\alpha cr}$	Eq. 11 Increments	AF_{B2}	Eq. 11 Increments
4A	1.162	2	1.174	3	1.312	4
	1.392	3	1.420	5	1.401	5
	1.743	5	1.798	6	1.775	6
	2.350	8	2.449	10	2.463	10
4B	1.356	3	1.425	5	1.426	5
	1.591	4	1.717	6	1.718	6
	1.937	6	2.159	8	2.161	8
	2.942	12	3.517	15	3.528	15
4C	1.424	3	1.514	5	1.500	5
	1.671	5	1.827	7	1.801	7
	2.033	7	2.302	9	2.251	9
	3.094	13	3.778	16	3.602	16
4D	1.188	2	1.199	3	1.193	3
	1.469	3	1.498	5	1.490	5
	1.935	6	1.995	7	2.013	8
	2.860	11	2.986	12	3.217	14
4E	1.160	2	1.188	3	1.192	3
	1.578	4	1.699	6	1.721	6
	1.943	6	2.164	8	2.212	9
	2.366	9	2.724	11	2.812	12
4F	1.154	2	1.171	3	1.138	3
	1.697	5	1.779	6	1.686	6
	2.253	8	2.402	10	2.225	9
	3.407	14	3.699	16	3.457	15

Appendix C (Results for Frames 5 through 15)

Frame	'Exact' AF	Actual Increments	$AF_{\alpha cr}$	Eq. 11 Increments	AF_{B2}	Eq. 11 Increments
5A	1.161	2	1.167	3	1.178	3
5B	1.811	5	1.872	7	2.022	8
6A	1.172	2	1.195	3	1.202	4
6B	2.586	10	2.980	12	3.682	16
7A	1.329	3	1.372	4	1.399	4
7B	1.948	6	2.112	8	2.326	9
8A	1.523	4	1.598	5	1.674	6
8B	2.290	8	2.536	10	3.106	13
9A	1.686	5	1.789	6	1.919	7
9B	2.510	10	2.808	12	3.567	15
10A	1.220	2	1.258	4	1.245	4
10B	1.755	5	1.919	7	1.988	7
11A	1.153	1	1.171	3	1.273	4
11B	2.739	11	3.500	15	4.182	18
12A	1.196	2	1.214	4	1.336	4
12B	3.373	14	3.817	17	6.401	30
13A	1.197	2	1.206	4	1.234	4
13B	2.919	11	3.088	13	4.408	20
14A	1.170	2	1.177	3	1.204	4
14B	2.002	6	2.056	8	2.398	9
15A	1.180	2	1.186	3	1.218	4
15B	1.808	5	1.836	7	2.076	8